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GEOPHYSICS

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GEOPHYSICS AS A SCIENCE*

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"Geophysics" means "Physics of the Earth." While in physics one tries to eliminate the effects of the gravitational, electric, and magnetic fields of the earth, in geophysics these fields, their properties and effects, and the structure and properties of parts of the earth are studied.

The science of geophysics can be divided into special fields by using either the subdivisions of physics, as mechanics, electricity, optics etc., or the three major parts of the earth: solid body, hydrosphere, and atmosphere. Moreover, in many cases we have "general" . . . as well as "economic" or "applied" . . . , problems. In the following tabulation which gives some major problems of geophysics by using the divisions just mentioned, general and applied problems are given under (a) and (b), respectively.

	Solid Body	*Hydrosphere*	*Atmosphere*
Mechanics	(a) Forces and stresses; gradual and sudden movements; earthquakes, earthquake waves, elastic and viscous movements; tides; movements of the poles; figure; density; volcanism (belongs also in other sections and geochemistry); mechanical effects of ice, water, wind. (b) Reduction of earthquake damage; seismic prospecting	(a) Tides, waves, currents; hydrology (b) Investigation and prediction of tides and currents for navigation and fisheries; hydrology; echo sounding	(a) Tides, waves including sound; currents (b) Weather forecasting
Gravitation	(a) Gravity; layering; pressure; isostasy; sedimentation (b) Use of pendulum, torsion balance, etc., in prospecting	(a) Layering; sedimentation	(a) Distribution of gases; layering

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	Solid Body	*Hydrosphere*	*Atmosphere*
Electricity	(a) Electric currents and electric waves (b) Electric prospecting	(a) Electric phenomena	(a) Electric phenomena; ionosphere; aurora
Magnetism	(a) Earth's magnetism (b) Magnetic prospecting	(b) Compass; magnetic charts	
Optics		(a) Color and transparency of lakes, oceans	(a) Meteorological optics, halos etc.; color of sky; polarization; turbidity; colors of clouds. (b) Visibility for aviation.
Composition of matter	(a) Composition of the earth; radioactivity; state of the interior	(a) Radioactivity of hydrosphere; salt content	(a) Composition of the atmosphere, ozone, etc.
Heat	(a) Temperature in the earth and its changes; crystalization and melting (b) Thermal prospecting	(a) Temperature in lakes, rivers and oceans; glaciers; icebergs; thermal currents	(a) Thermodynamics of the atmosphere; temperature; climates (include other factors) (b) Climatology

The entire science of Geophysics as sketched above is seldom considered as a separate division of knowledge, comparable to geology or astronomy. One reason for this attitude is the fact that no one person can be familiar with all parts of the geophysical field. Usually, geophysicists either specialize in problems concerning the solid earth, or in oceanography, hydrology or meteorology; some are more familiar with general geophysics, others with economic problems; some prefer questions of mechanics, others of electricity. However, there are many problems which occur in wholly different sections of geophysics, but which require almost the same theoretical treatment. The same equations are to be used for the propagation of earthquake waves, explosion waves in seismic prospecting, in echo sounding in the ocean, and for the investigation of the so called "anormal sound waves" in the atmosphere. Many problems of oceanography and meteorology are similar. In several respects the ocean is an image of the atmosphere with a disturbed layer near the surface of the earth, followed by a layer with decrease in temperature and finally a relatively quiet layer (stratosphere).

Geophysics is a very young science, and this is another reason for

its not being considered as a separate division of knowledge. Meteorology separated relatively early from the science of the "meteors." Oceanography has developed in most countries under the national departments of commerce to reduce the dangers of navigation. Seismological observatories can be found today as a part of geological, geographical, astronomical, meteorological, mining, and engineering observatories and departments. As a rare exception, the Saint Louis University may be quoted, where geology belongs to the department of geophysics.

The relationship between geology and geophysics has been discussed frequently. There can be little doubt, however, that geophysics and geology must cooperate to get a maximum of results. This is a fact from the viewpoint of geology as well as of geophysics, and for problems concerning the structure of the earth's crust as well as for investigations concerning local problems of economic importance.

FUNDAMENTAL RELATIONS IN SEISMOMETRY*

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ABSTRACT

The equations for dynamical equilibrium of a geophone are derived by means of the Lagrange method and applied to the particular cases of a moving armature geophone, moving conductor geophone, and electrostatic geophone. The galvanometer for seismic recording is usually of the moving coil type and its equations are similar to those of a moving conductor geophone.

The magnetic type geophone as well as the galvanometer consist essentially of two dynamical systems characterized by an asymmetric coupling: "the transducing resistance."

A brief analysis is given of the problem of determining the response of a seismograph to a given earth motion.

INTRODUCTION

This investigation has been made in connection with the development of instruments for geophysical prospecting. It is attempted here to discuss several problems involved in the registration of earth motion and to present certain seismometric concepts which have been used by the authors in their work.

A series of six papers is intended for presentation:

1. Fundamental Relations in Seismometry.
2. Equivalent Electrical Networks of Some Seismographs.
3. Geophone Measurements.
4. Distortionless Recording of the Earth Motion.
5. Intentional Distortion in Seismographs for Reflection Prospecting.
6. Non-linear Relationships in Geophones.

Of these the first topic, referring to the fundamental relations in seismometry, is discussed in the present paper.

Before proceeding, it will be convenient to define a "seismograph" as an arrangement including means for detecting and recording earth vibrations, and a "geophone" as an electromechanical transducer actuated by power from the earth vibrations and supplying power to a recording system, the wave form in the recording system corresponding to the wave form of the earth vibrations. The term "seismograph" was introduced by Palmieri and together with "seismometer" was used for the first time in 1841 (Reference 1). This marks

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the beginning of modern seismological terminology, though an earthquake detector was already known and used eighteen centuries ago by a Chinese scholar Chang Hêng. The term "geophone" is of relatively recent origin and designates an apparatus developed by the French during the world war for listening underground. Within recent years "geophone" has been consistently used to designate a seismic detector of electric type used for seismic prospecting and in this paper the meaning of this term has not been changed.

Two geophone types shall be investigated: a magnetic and an electrostatic geophone. A magnetic geophone may be either of the dynamic (moving conductor) or of the moving armature type. The latter type involves the vibration of a ferromagnetic circuit.

ACKNOWLEDGMENT

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(1) EQUATIONS FOR DYNAMICAL EQUILIBRIUM OF A GEOPHONE

A geophone consists of two dynamical systems mutually coupled one to another: the mechanical and the electrical system. The constitution of the mechanical system can be described in terms of inertia, resistance, and compliance, and that of the electrical system in terms of inductance, resistance, and capacitance. Two variables will be used to describe the motion: the mechanical displacement s and the electrical displacement q, or the mechanical displacement s and the electrical current i.

The formulation of dynamic equilibrium conditions can be derived by means of Lagrange equations (Reference 2) and expressed in terms of the total instantaneous kinetic energy T, potential energy V, and dissipation function D.

The total kinetic energy T of the system consists of two terms

$$T = T_m + T_e \tag{1}$$

of which the first term expresses the energy due to the motion in the mechanical system,

$T_m = \frac{1}{2} m_m \dot{s}^2$ (m_m = inertia coefficient in the mechanical system), (2)

and the second term T_e corresponds to the energy stored in the magnetic field (if present).

The total potential energy V includes two terms:

$$V = V_m + V_e, \tag{3}$$

where V_m depends upon the mechanical displacement:

$$V_m = V_m(s) \tag{4}$$

and V_e depends upon the electrical displacement:

$$V_e = V_e(q). \tag{5}$$

Consider the function V_m and assume that $V_m = 0$ when $s = 0$. Since $s = 0$ will be chosen as a point of equilibrium then $\frac{dV_m}{ds} = 0$ when $s = 0$. Consequently, if the function (4) be expanded in Maclaurin series the term containing the first power of s will be absent and

$$V_m = \frac{1}{2!}\left(\frac{d^2V_m}{ds^2}\right)_{s=0} s^2 + \frac{1}{3!}\left(\frac{d^3V_m}{ds^3}\right)_{s=0} s^3 + \cdots. \tag{6}$$

In this expression the term $\frac{d^2V}{ds^2} = \frac{1}{C_m}$, where C_m is the compliance of the spring. In the further discussion it shall be assumed that the springs obey Hooke's law and consequently the terms consisting of powers higher than the second in (6) shall be neglected.

The potential energy stored in the electrical system is expressed as

$$V_e = \frac{1}{2C} q^2, \tag{7}$$

where C is the capacitance of the electrical system.

To take into account frictional forces, a dissipation function is introduced which expresses the amount of energy converted into heat. The dissipation function consists of two terms

$$D = D_m + D_e, \tag{8}$$

where $D_m = D_m(\dot{s})$ is the mechanical dissipation function (9)

and $D_e = D_e(i)$ is the electrical dissipation function. (10)

Express the function $D_m(\dot{s})$ in Maclaurin series:

$$D_m = \frac{1}{2}\left(\frac{d^2D_m}{d\dot{s}^2}\right)_{\dot{s}=0}\dot{s}^2 + \frac{1}{4}\left(\frac{d^4D_m}{d\dot{s}^4}\right)_{\dot{s}=0}\dot{s}^4 + \cdots . \tag{11}$$

The odd powers of $\dot{s}$ cannot be present in this expression since

$$D_m(\dot{s}) = D_m(-\dot{s}) \tag{12}$$

i.e., the dissipation remains the same if the motion changes its direction. In the expression (11) the term

$$\frac{d^2D_m}{d\dot{s}^2} = r_m \tag{13}$$

designates the mechanical resistance factor and is determined by the side slip between a moving mass and a viscous medium. The terms containing higher powers of $\dot{s}$ are due to the paddle effect of the moving mass and shall be neglected in this discussion. (To be described in a later paper.)

The electrical dissipation function $D_e(i)$ has the electrical resistance factor R as a coefficient and when differentiated with respect to the current gives the dissipational force against which the work is done in the electrical system:

$$D_e(i) = \frac{Ri^2}{2}. \tag{14}$$

As a result of the application of the Lagrange method the equations of the motion in a geophone are automatically deduced from the following equations:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{s}} - \frac{\partial(T-V)}{\partial s} + \frac{\partial D}{\partial \dot{s}} = f, \tag{15}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial(T-V)}{\partial q} + \frac{\partial D}{\partial \dot{q}} = e. \tag{16}$$

The equation (15) expresses the dynamic equilibrium condition in the mechanical system of the geophone, the equation (16) expresses the equilibrium condition in the electrical system, and f, e designate the respective values of the external mechanical force and the external electrical force.

In the above formulas T, V and D are expressed in joules, s in centimeters, $\dot{s}$ in cm./sec., q in coulombs, i in amperes, m_m in grams $\times 10^{-7}$, r_m in (dyne sec./cm.) $\times 10^7$, C_m in (cm./dynes) $\times 10^7$, C in far-

ads, R in ohms, the inductance L in henrys, f in dynes $\times 10^{-7}$, e in volts.

(2) MOVING ARMATURE GEOPHONE

Fig. 1 shows a moving armature geophone adapted for seismic prospecting. It consists essentially of two permanent magnets elastically supported by two springs attached to the case and of an armature placed in the air gap between the magnets and rigidly connected to the case. During the arrival of seismic waves a relative motion is produced between the armature and the magnets and the reluctance of the various magnetic paths involved in the structure is varied. As a result of electromagnetic induction the varying magnetic flux causes a current to flow in a coil surrounding the armature.

A schematic representation of the geophone is shown in Fig. 2. The magnets are shown to be displaced from their neutral position since the line M_1N_1 which is equidistant from the respective pole pieces does not coincide with the armature axis M_2N_2. The relative position of the magnets with regard to the armature is determined by the distance s between the lines referred to.

The dynamical equilibrium conditions will be determined by the Lagrange method and consequently the kinetic energy, potential energy and the dissipation function will be successively calculated.

The kinetic energy of the geophone includes the following components:

$$T = \tfrac{1}{2} m_m \dot{s}^2 + T_{e1} + T_{e2} + T_{e3}, \tag{17}$$

where m_m is the suspended mass (permanent magnets),

T_{e1} is the energy stored in the permanent magnetic field,

T_{e2} is the magnetic energy due to the current i in the armature coil, and

T_{e3} is the magnetic energy due to the reaction of the current i upon the permanent magnetic field.

Certain idealizing assumptions are made here concerning the path of the magnetic lines and, accordingly, the magnetic energy is assumed to be contained only within the air gaps and in the magnetic material and not to be contained in the outside space.

Let:

$\mathfrak{F}$ = the effective magnetomotive force of each magnet,

$\mathfrak{R}$ = the reluctance of each magnet,

$\mathfrak{R}_a$ = the reluctance of the armature,

$\mathfrak{R}_g$ = the reluctance of each air gap when $s = 0$ (neutral position),

FIG. 1. Moving armature geophone.

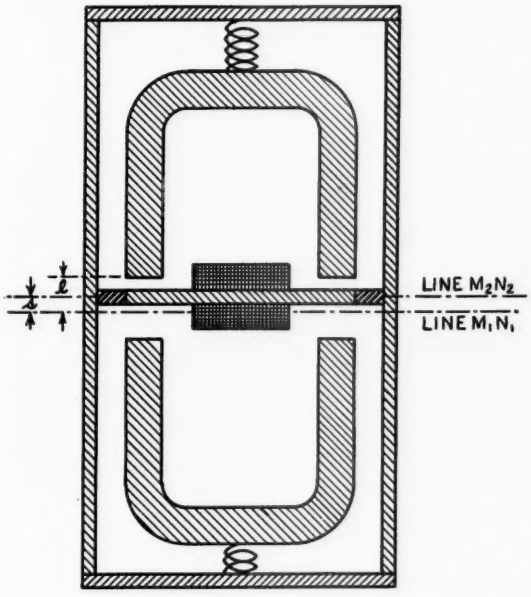

FIG. 2. Moving armature geophone.

ϕ_1 = the magnetic flux through the upper magnet,
ϕ_2 = the magnetic flux through the lower magnet,
ϕ_a = the magnetic flux through the armature, and
$\mathfrak{F}_a$ = the magnetomotive force created by the current in the armature coil.

Here the $\mathfrak{F}$'s are expressed in gilberts, the $\mathfrak{R}$'s in oersteds, and the ϕ's in maxwells.

From Fig. 2 it is also seen that:

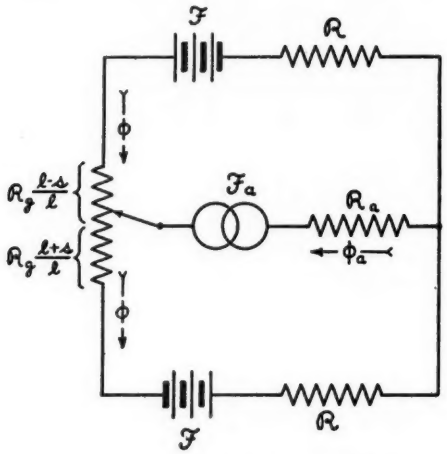

FIG. 3. Magnetic circuit of a moving armature geophone.

$\mathfrak{R}_g \dfrac{l-s}{l}$ = the reluctance of the two air gaps (in series) above the armature,

$\mathfrak{R}_g \dfrac{l+s}{l}$ = the reluctance of the two air gaps (in series) below the armature.

The permanent magnets are designed primarily to give the maximum possible flux density for a given magnetomotive force. This can only be accomplished by reducing the relative value of the reluctance of the magnetic material when compared to the reluctance of the air gap. Since the reluctance of the air gap is independent of the flux density, and the reluctance of the magnetic material increases with the flux density, the efficient design of the magnetic structure will have all the iron working at a flux density below saturation. It is

assumed here that the working region is contained in the rectilinear part of the magnetization curve and consequently the parameters of the magnetic circuit are assumed to be linear.

The magnetic relationships in the arrangement of Fig. 2 are translated into electrical relationships by means of the equivalent electric network in Fig. 3, in which the magnetomotive forces have been substituted by electromotive forces, reluctances by resistances and magnetic flux by electric current.

Applying Kirchoff's laws the following relations are deduced:

$$\phi_a = \phi_1 - \phi_2 \tag{18}$$

$$\mathfrak{F} = \phi_1 \left[\mathcal{R}_g \frac{l-s}{l} + \mathcal{R} \right] + \phi_a \mathcal{R}_a \tag{19}$$

$$\mathfrak{F} = \phi_2 \left[\mathcal{R}_g \frac{l+s}{l} + \mathcal{R} \right] - \phi_a \mathcal{R}_a \tag{20}$$

where

$$\phi_1 = \frac{\mathfrak{F}\left[\mathcal{R}_g \frac{l+s}{l} + \mathcal{R} + 2\mathcal{R}_a \right]}{\left[\mathcal{R}_g \frac{l+s}{l} + \mathcal{R} + \mathcal{R}_a \right] \left[\mathcal{R}_g \frac{l-s}{l} + \mathcal{R} + \mathcal{R}_a \right] - \mathcal{R}_a^2} \tag{21}$$

$$\phi_2 = \frac{\mathfrak{F}\left[\mathcal{R}_g \frac{l-s}{l} + \mathcal{R} + 2\mathcal{R}_a \right]}{\left[\mathcal{R}_g \frac{l+s}{l} + \mathcal{R} + \mathcal{R}_a \right] \left[\mathcal{R}_g \frac{l-s}{l} + \mathcal{R} + \mathcal{R}_a \right] - \mathcal{R}_a^2} \tag{22}$$

$$\phi_a = \frac{2\mathfrak{F}\mathcal{R}_g \frac{s}{l}}{\left[\mathcal{R}_g \frac{l+s}{l} + \mathcal{R} + \mathcal{R}_a \right] \left[\mathcal{R}_g \frac{l-s}{l} + \mathcal{R} + \mathcal{R}_a \right] - \mathcal{R}_a^2}. \tag{23}$$

The kinetic energy components can now be successively determined: *Energy due to the permanent magnetism* is expressed by the following formula:

$$T_{e1} = \left[\phi_1^2 \left(\mathcal{R} + \mathcal{R}_g \frac{l-s}{l}\right) + \phi_a^2 \mathcal{R}_a + \phi_2^2 \left(\mathcal{R} + \mathcal{R}_g \frac{l+s}{l}\right)\right] \times \frac{10^{-7}}{8\pi}. \quad (24)$$

The expression T_{e1} is a function of s and can be expanded in Maclaurin series

$$T_{e1} = (T_{e1})_0 + \left(\frac{d}{ds} T_{e1}\right) s + \frac{1}{2!} \left(\frac{d^2}{ds^2} T_{e1}\right) s^2 + \cdots \quad (25)$$

in which the powers of s higher than the first shall be neglected.

The energy stored by the electrical current i can be determined as follows:

The magnetomotive force created by the current i is

$$\mathfrak{F}_a = 0.4\pi ni. \quad (26)$$

The impedance of the corresponding magnetic circuit consists of $\mathcal{R}_a$ in series with a combination of $\mathcal{R} + \mathcal{R}_g \frac{l-s}{l}$ in parallel with $\mathcal{R} + \mathcal{R}_g \frac{l+s}{l}$ and can be expressed as follows:

$$\mathcal{R}_i = \mathcal{R}_a + \frac{\left[\mathcal{R} + \mathcal{R}_g \frac{l-s}{l}\right]\left[\mathcal{R} + \mathcal{R}_g \frac{l+s}{l}\right]}{2(\mathcal{R} + \mathcal{R}_g)}. \quad (27)$$

Consequently the flux created by the current i is

$$\phi_i = \frac{0.4\pi ni}{\mathcal{R}_i}, \quad (28)$$

and the energy stored in the magnetic field of the current i,

$$T_{e2} = \frac{0.4\pi ni\phi_i}{8\pi} \times 10^{-7}. \quad (29)$$

Let

$$T_{e2} = \tfrac{1}{2} L_1 i^2 \quad (30)$$

where

$$L_1 = \frac{n\phi_i}{i} \times 10^{-8}. \tag{31}$$

Using Maclaurin's expansion the expression (30) can be written as follows:

$$T_{e2} = \frac{1}{2} i^2 \left[L + \left(\frac{dL_1}{ds}\right)_0 s + \frac{1}{2!} \left(\frac{d^2L_1}{ds^2}\right) s^2 + \cdots \right] \tag{32}$$

where

$$L = (L_1)_{s=0}.$$

The magnetic energy due to the reaction of the current i upon the magnetic field is expressed as follows:

$$T_{e3} = \frac{ni\phi_a}{2} \times 10^{-7}. \tag{33}$$

Write

$$T_{e3} = Ki \qquad \text{where} \qquad K = \frac{n\phi_a}{2} \times 10^{-8} \tag{34}$$

and express T_{e3} in the form of MacLaurins series:

$$T_{e3} = i \left[\left(\frac{dK}{ds}\right)_0 s + \frac{1}{2!} \left(\frac{d^2K}{ds^2}\right)_0 s^2 + \frac{1}{3!} \left(\frac{d^3K}{ds^3}\right)_0 s^3 + \cdots \right] \tag{35}$$

where

$$(K)_{s=0} = 0, \qquad \text{since} \qquad (\phi_a)_{s=0} = 0. \tag{36}$$

Thus the total kinetic energy of the system can be expressed as follows:

$$T = \frac{1}{2} m_m \dot{s}^2 + (T_{e1})_0 + \left(\frac{dT_{e1}}{ds}\right)_0 s + \frac{1}{2} Li^2 + \frac{1}{2} \left(\frac{dL_1}{ds}\right)_0 i^2 s + \left(\frac{dK}{ds}\right)_0 is. \tag{37}$$

Since there is no capacity in the electrical system of the geophone, the term $V_e = 0$ and consequently the potential energy function comprises only the term V_m.
Thus:

$$V = V_m = \frac{1}{2C_m} s^2 \tag{38}$$

where C_m is the compliance of the spring.

The dissipation function is

$$D = \tfrac{1}{2} r_m \dot{s}^2 + \tfrac{1}{2} R i^2 \tag{39}$$

where r_m is the mechanical resistance factor and R is the resistance of the armature coil.

We shall make use now of the Lagrange method by substituting the above values of energy in (15) and (16), and taking as generalized coordinates s and i. Then

$$m_m \ddot{s} - \left(\frac{dT_{el}}{ds}\right)_0 - \frac{1}{2}\left(\frac{dL_1}{ds}\right)_0 i^2 - \left(\frac{dK}{ds}\right)_0 i + r_m \dot{s} + \frac{1}{C_m} s = f. \tag{40}$$

Assume that

$$\left(\frac{dT_{el}}{ds}\right)_0 = 0; \quad \left(\frac{dL_1}{ds}\right)_0 = 0; \qquad \text{and put}$$

$$\left(\frac{dK}{ds}\right)_0 = \left(\frac{d\phi_a}{ds}\right)_{s=0} \frac{n}{2} 10^{-8} = K_{gm}.$$

Consequently

$$m_m \ddot{s} + r_m \dot{s} + \frac{s}{C_m} - K_{gm} i = f. \tag{41}$$

The expression (41) gives the conditions of dynamic equilibrium in the mechanical system of the geophone.

The conditions of dynamic equilibrium in the electrical system of the geophone can be derived from (16).
Then

$$\frac{d}{dt}\left[Li + \left(\frac{dL_1}{ds}\right)_0 is + \left(\frac{dK}{ds}\right)_0 s\right] + Ri = e \tag{42}$$

which by taking into account the assumptions made above is reduced to the form

$$L\frac{di}{dt} + Ri + K_{gm}\dot{s} = e. \tag{43}$$

The expressions (41) and (43) constitute the fundamental equations of a moving armature geophone.

Consider now the actual conditions occurring during the arrival of seismic waves. Adopt a fixed reference axis and express the earth motion by the function

$$w = w(t). \tag{44}$$

Then if z designates the position of the mass with regard to the fixed axis, the force of inertia is expressed as $m\ddot{z}$, the potential energy is determined by the relative displacement $s=z-w$ and the force of friction will depend upon the relative velocity $\dot{s}=\dot{z}-\dot{w}$. Consequently the equation of motion in the mechanical system will assume the following form

$$m_m\ddot{z} + r_m\dot{s} + \frac{1}{C_m}s - K_{gm}i = 0 \tag{45}$$

and since $z=s+w$

$$m_m\ddot{s} + r_m\dot{s} + \frac{1}{C_m}s - K_{gm}i = -m_m\ddot{w}. \tag{46}$$

The reference directions of $-m_m\ddot{w}$, $\dot{s}$, e, i have been made symmetrical. That is the product $-(m_m\ddot{w})\dot{s}$ when positive represents the actual instantaneous mechanical power input and the product ei if positive represents the actual instantaneous electrical power input. Since, however, in a geophone the mechanical motion is steadily converted into electrical i.e., the energy flow is unidirectional, possibly a more familiar picture would be given by changing the reference axis for the current i.

$$i_1 = -i \tag{47}$$

so that the product ei when positive expresses the power output of the geophone.

However, in order to generalize the analysis more easily the sign convention as expressed by the equations (43), (46) has been retained.

(3) MOVING CONDUCTOR GEOPHONE

The moving conductor geophone does not differ essentially from the moving armature geophone. In both types the principle of electromagnetic induction is utilized and similar differential equations are involved. A difference between the moving conductor geophone and the

moving armature geophone is the presence of a negative stiffness component in the latter (due to the reaction between the permanent magnet and the moving armature). This difference has importance from the standpoint of design only. In the present discussion the resultant stiffness shall be assumed to be constant which is approximately true for small motions. For larger displacements the non-linearity in stiffness should be taken into account.

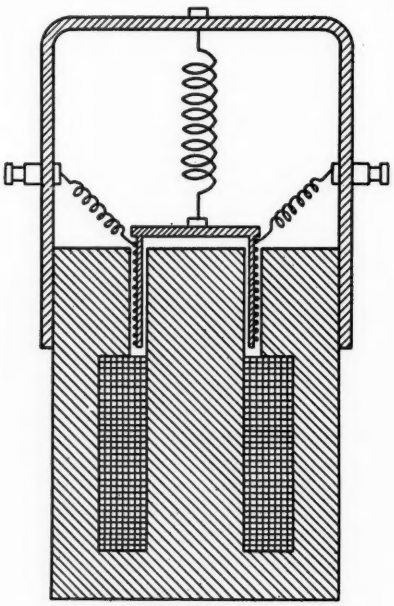

FIG. 4. Moving conductor geophone.

A moving conductor geophone adapted for seismic prospecting is shown in Figure 4 (Reference 3). It consists of a coiled conductor supported elastically by the case and of an electromagnet rigidly attached to the case. The relative motion between the electromagnet and the suspended conductor induces in the latter electrical currents which represent the earth vibrations.

The dynamical conditions of equilibrium are established as previously by making use of the kinetic energy, the potential energy and the dissipation function.

The kinetic energy of the geophone is

$$T = \tfrac{1}{2}m_m\dot{s}^2 + \tfrac{1}{2}L_0\dot{i}_0^2 + \tfrac{1}{2}L\dot{i}^2 + \tfrac{1}{2}M\dot{i}_0\dot{i}, \qquad (48)$$

where L_0 is the self inductance of the electromagnet winding,
L_1 is the self inductance of the suspended coil,
M is the mutual inductance between the electromagnet winding and the suspended coil,
i_0 is the current in the electromagnet winding (i_0 is assumed to be constant),
$i = \dot{q}$ is the current through the suspended coil.
m_m is the mass of the electromagnet.

Here the mutual inductance depends upon the displacement, i.e.,

$$M = M(s) \tag{49}$$

and the inductances L_0, L_1 are constant.

Express the coefficient of mutual inductance in form of Maclaurin's expansion

$$M = M_0 + \left(\frac{dM}{ds}\right)_0 s + \frac{1}{2!}\left(\frac{d^2M}{ds^2}\right)s^2 + \cdots \tag{50}$$

and neglect in this expansion the powers of s higher than one.

The potential energy of the geophone is:

$$V = \frac{1}{2C_m} s^2 \tag{51}$$

where C_m is the compliance of the spring, and the dissipation function

$$D = \tfrac{1}{2} r_m \dot{s}^2 + \tfrac{1}{2} R i^2 + \tfrac{1}{2} R_0 i_0^2 \tag{52}$$

where r_m is the mechanical resistance factor.
R is the resistance of the suspended coil.
R_0 is the resistance of the electromagnet coil.

Substitute now (48), (51), (52) in (15) and (16). Then

$$m_m \ddot{s} + r_m \dot{s} + \frac{1}{C_m} s - K_{gm} \dot{q} = f \tag{53}$$

$$L \frac{di}{dt} + Ri + K_{gm} \dot{s} = e \tag{54}$$

where

$$K_{gm} = \frac{1}{2}\left(\frac{dM}{ds}\right)_0 i_0.$$

If w expresses the impressed wave motion, then the equation (53) assumes the following form:

$$m_m\ddot{s} + r_m\dot{s} + \frac{1}{C_m}s - K_{gm}\dot{q} = -m_m\ddot{w}. \tag{55}$$

It is observed that the equations (54), (55) representing the moving conductor geophone are similar to the equations (43) and (46) representing the moving armature geophone. However, in the moving armature type

$$K_{gm} = \frac{n}{2}\left(\frac{d\phi_a}{ds}\right)_0 \times 10^{-8} \tag{56}$$

while in the moving conductor type

$$K_{gm} = \frac{1}{2}\left(\frac{dM}{ds}\right)_0 \times i_0. \tag{57}$$

The coefficient K_{gm} will be designated as "transducing resistance."

(4) ELECTROSTATIC GEOPHONE

A schematic structure of an electrostatic geophone is shown in Fig. 5. It includes a mechanical and an electrical system and the coupling between the two is provided by means of a condenser ar-

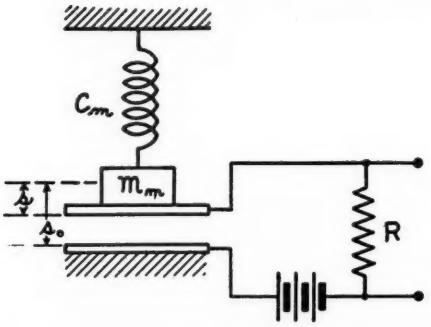

FIG. 5. Electrostatic geophone.

ranged in such a manner that the impressed earth motion causes a relative displacement between the plates of the condenser and induces electrostatically a varying electromotive force in the electrical system.

The dynamic conditions of equilibrium are determined as above

by making use of the kinetic energy, the potential energy and the dissipation function.

The kinetic energy is contained only in the mechanical system

$$T = \tfrac{1}{2} m_m \dot{s}^2 \tag{58}$$

where s = the relative displacement of the moving condenser plate from its position of equilibrium; and m_m is the suspended mass.

The potential energy is contained partly in the mechanical system

$$V_m = \frac{1}{2C_m} s^2 \tag{59}$$

where C_m is the compliance of the spring and partly in the electrical system

$$V_e = \frac{1}{2C} q_1^2 \tag{60}$$

where C is the capacitance of the condenser and q_1 is its charge. Let the distance between the condenser plates at the position of equilibrium ($s=0$) be s_0. Then the capacitance C varies with s according to the function

$$C = \frac{C_0 s_0}{s_0 - s} \tag{61}$$

the potential energy stored in the condenser is

$$V_e = \frac{1}{2C_0} \frac{(s_0 - s)}{s_0} q_1^2 \tag{62}$$

and consequently the total potential energy of the geophone can be expressed as follows:

$$V = \frac{1}{2C_m} s^2 + \frac{1}{2C_0} \frac{(s_0 - s)}{s_0} q_1^2. \tag{63}$$

The dissipation function is equal to:

$$D = \tfrac{1}{2} r_m \dot{s}^2 + \tfrac{1}{2} R \dot{q}_1^2. \tag{64}$$

Substitute now (58), (63), (64) in the Lagrange equations. Then:

$$m_m \ddot{s} + r_m \dot{s} + \frac{s}{C_m} - \frac{1}{2C_0 s_0} q_1^2 = f \tag{65}$$

$$R\dot{q}_1 + \frac{1}{C_0} q_1 - \frac{1}{C_0 s_0} q_1 s = e. \tag{66}$$

Put

$$q_1 = q_0 + q \tag{67}$$

where q_0 is the steady average component of q_1 and q is the varying component of q_1. Thus

$$\dot{q}_1 = \dot{q} \tag{68}$$

$$q_1^2 = q_0^2 + 2q_0 q + q^2. \tag{69}$$

Assuming that

$$q \ll q_0 \tag{70}$$

the alternating component of q_1^2 is substantially equal to $2q_0 q$.

Disregarding the steady components the equations (65), (66) are reduced to the following form:

$$m_m \ddot{s} + r_m \dot{s} + \frac{1}{C_m} s - \frac{q_0}{C_0 s_0} q = f \tag{71}$$

$$R\dot{q} + \frac{1}{C_0} q - \frac{q_0}{C_0 s_0} s = e. \tag{72}$$

The equation (71) may be transformed into the following form:

$$m_m \ddot{s} + r_m s + \frac{1}{C_m} s - \frac{q_0}{C_0 s_0} q = - m_m \ddot{w}. \tag{73}$$

Thus the equations (72) and (73) express the dynamic equilibrium conditions of a capacity geophone.

(5) OSCILLOGRAPH

The oscillograph is used to produce a photographic record of seismic waves. The essential constituents of the oscillograph are: galvanometer, camera, and an appropriate optical system. The galvanometer is usually of the moving coil type ("d'Arsonval type") and consists of an electromagnet and of a coil of wire through which current may flow (see Fig. 6). The coil is arranged to rotate and its inertia is usually very small in order that it may follow the rapid variation of the impressed current. A small mirror is cemented to the coil, which is immersed in a damping liquid and placed between the poles of the electromagnet. Fig. 7 shows a galvanometer designed to satisfy

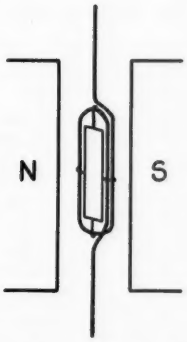

FIG. 6. Moving coil of a galvanometer.

the particular needs of seismic recording. It possesses a simple compact structure wherein all the vibrating elements are exactly the same and located in the same magnetic fields (Reference 4).

In general, the galvanometer is an electromechanical transducer which possesses essentially the same physical elements as the geophone. In a geophone the mechanical force originated by the wave motion is translated into an electrical effect while in a galvanometer the process is reversed, i.e., a varying electrical effect is translated into a mechanical motion which in turn is directly recorded on a photographic film.

Let

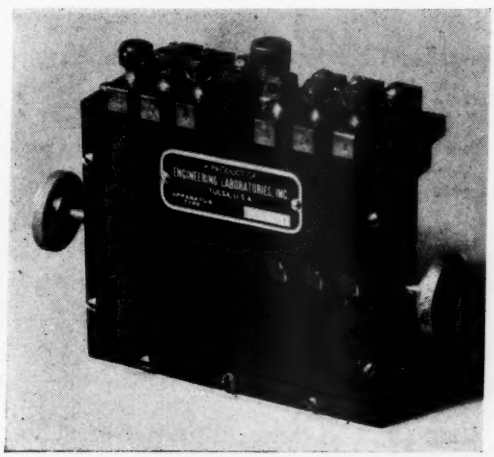

FIG. 7. Galvanometer.

I_{mg} be the moment of inertia of the moving system of the galvanometer,

r_{mg} the damping constant in the moving system of the galvanometer,

C_{mg} the mechanical compliance of the galvanometer spring,

θ_g the angular deviation of the galvanometer,

ϕ_g the flux through the galvanometer coil when the coil is at right angles to the magnetic flux,

i_g the current through the galvanometer coil,

L_g the inductance of the galvanometer coil,

R_g resistance of the galvanometer coil,

n_g number of turns in the galvanometer coil.

Here I_{mg} is expressed in gram $cm^2 \times 10^{-7}$, θ_g in radians, $\dot{\theta}_g$ in radian/sec., r_{mg} in $\frac{\text{dyne cm. sec.}}{\text{radian}} \times 10^{-7}$, C_{mg} in $\frac{\text{radian}}{\text{dyne cm}} \times 10^{7}$.

The neutral position of the galvanometer coil corresponds to $\theta_g = 0$.

The equations of the galvanometer can be deduced by means of the Lagrange method in the same manner as the equations of the geophone. Consequently, the kinetic energy, the potential energy and the dissipation function shall be subsequently determined.

The total kinetic energy is equal to:

$$T_g = \tfrac{1}{2} I_{mg} \dot{\theta}_g^2 + T_{mg} + \tfrac{1}{2} n_g \phi_g i_g \sin \theta_g \times 10^{-8} + \tfrac{1}{2} L_g i_g^2 \qquad (74)$$

where

$\frac{1}{2} I_{mg} \dot{\theta}_g^2$ is the kinetic energy stored in the inertia of the rotating mass,

T_{mg}, the energy stored in the field of the electromagnet (T_{mg} = const.)

$\frac{1}{2} n_g \phi_g i_g \sin \theta_g \times 10^{-8}$, the energy due to the interaction between the current i_g and flux ϕ_g passing through the coil, and

$\frac{1}{2} L_g i_g^2$, the magnetic energy stored in the inductance of the coil.

The potential energy of the galvanometer system is

$$V_g = \frac{1}{2C_{mg}} \theta_g^2. \qquad (75)$$

The dissipation function can be expressed as follows

$$D_g = r_{mg} \dot{\theta}_g^2 + R_g i_g^2 + D_0 \qquad (76)$$

where

D_0 is the dissipation function corresponding to the energy loss in the coil of the electromagnet (D_0=const.).

Applying Lagrange equations the conditions of dynamic equilibrium are determined as follows:

$$I_{mg}\ddot{\theta}_g + r_{mg}\dot{\theta}_g + \frac{1}{C_{mg}}\theta - \tfrac{1}{2}n_g\phi_g i_g \cos\theta_g \times 10^{-8} = 0 \qquad (77)$$

$$L_g\frac{di_g}{dt} + R_g i_g + \tfrac{1}{2}10^{-8}n_g\phi_g \cos\theta_g\dot{\theta}_g = e_g. \qquad (78)$$

Assume that the angular deviations are small so that $\cos\theta_g \approx 1$. Then:

$$I_{mg}\ddot{\theta}_g + r_{mg}\dot{\theta}_g + \frac{1}{C_{mg}}\theta_g - K_g i_g = 0 \qquad (79)$$

$$L_g\frac{di_g}{dt} + R_g i_g + K_g\dot{\theta}_g = e_g \qquad (80)$$

where

$$K_g = \frac{n_g\phi_g}{2} \times 10^{-8}.$$

(6) MATHEMATICAL ANALYSIS OF SEISMIC REGISTRATION

Every actual seismographic system distorts the impressed earth motion; that is to say, the recorded motion regarded as a time function differs in shape from the impressed motion.

The present problem consists in determining the response of a seismograph to a given impressed ground motion. Since the magnitudes which are dealt with are of an essentially transient character the most convenient method of attack appears to be the one based upon Heaviside's Symbolic Calculus (Reference 5).

Assume that the earth vibrations $w(t)$ are impressed upon a magnetic geophone and the output terminals of the geophone are connected to a galvanometer. The problem consists in expressing the galvanometer motion θ as an explicit function of w.

Let e_p, i_p, s_p, w_p, e_{gp}, i_{gp}, θ_{gp} be functions of a variable p which are related to the functions e, i, s, w, e_g, i_g, θ_g by the following Laplacian transforms:

$$\frac{e_p}{p} = \int_0^\infty e\epsilon^{-pt}dt \tag{81-a}$$

$$\frac{i_p}{p} = \int_0^\infty i\epsilon^{-pt}dt \tag{81-b}$$

$$\cdots\cdots \text{etc.}$$

and be designated as "image functions" of e, i, s, w, e_g, i_g, θ_g respectively.

Assume that at a given location at which the geophone is placed the earth was at rest prior to the instant $t=0$ and that at $t=0$ a train of waves arrived. Let at $t=0$, $w=w_0$, $(dw/dt)=(dw/dt)_0$.

By following the procedure indicated by van der Pol which consists in multiplying both sides of a differential equation by $\epsilon^{-pt}dt$, integrating the result with respect to t from zero to infinity, and using the notation as shown in (81) the equations (43), (46) assume the following form:

$$Z_m(p)\dot{s}_p - K_{gm}i_p = m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p \tag{82}$$

$$Z_e(p)i_p + K_{gm}\dot{s}_p = e_p \tag{83}$$

or in matrix notation (Reference 6).

$$\begin{Vmatrix} Z_m(p) & -K_{gm} \\ K_{gm} & Z_e(p) \end{Vmatrix} \times \begin{Vmatrix} \dot{s}_p \\ i_p \end{Vmatrix} = \begin{Vmatrix} m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p \\ e_p \end{Vmatrix} \tag{84}$$

where

$Z_m(p)=m_m p+r_m+\dfrac{1}{pC_m}$ is the symbolic expression for "the mechanical impedance of the geophone."

$Z_e(p)=L_p+R$ is the symbolic expression for "the electrical impedance of the geophone."

K_{gm} is the transducing resistance of the geophone.

The relations (82) (83) can also be expressed in the form:

$$e_p = D(p)f_p - B(p)\dot{s}_p \tag{85}$$

$$i_p = C(p)f_p - A(p)\dot{s}_p \tag{86}$$

where:

$$f_p = m_m \left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p,$$

or, in matrix notation,

$$\left\| \begin{matrix} e_p \\ i_p \end{matrix} \right\| = \left\| \begin{matrix} D(p) & B(p) \\ C(p) & A(p) \end{matrix} \right\| \times \left\| \begin{matrix} f_p \\ -\dot{s}_p \end{matrix} \right\| \tag{87}$$

where:

$$\begin{aligned} D(p) &= -\frac{Z_e(p)}{K_{gm}}; & B(p) &= \frac{1}{K_{gm}} \\ C(p) &= -\frac{1}{K_{gm}}; & A(p) &= -\frac{Z_m(p)}{K_{gm}}. \end{aligned} \tag{88}$$

By applying the procedure of van der Pol already referred to, the galvanometer equation becomes:

$$Z_{eg}(p) i_{gp} + K_g \dot{\theta}_{gp} = e_{gp} \tag{89}$$

$$Z_{mg}(p) \dot{\theta}_{gp} - K_g i_{gp} = 0 \tag{90}$$

or in matrix notation:

$$\left\| \begin{matrix} Z_{eg}(p) & K_g \\ -K_g & Z_{mg}(p) \end{matrix} \right\| \times \left\| \begin{matrix} i_{gp} \\ \dot{\theta}_{gp} \end{matrix} \right\| = \left\| \begin{matrix} e_{gp} \\ 0 \end{matrix} \right\|. \tag{91}$$

Here

$Z_{eg}(p) = L_g p + R_g$ is the symbolic expression for the "electrical impedance of the galvanometer"

$Z_{mg}(p) = I_{mg} p + r_{mg} + \dfrac{1}{pC_{mg}}$ is the symbolic expression for the "mechanical impedance of the galvanometer," and

K_g is the transducing resistance of the galvanometer.

The relations (89) and (90) can also be expressed in the form:

$$0 = D_g(p) e_{gp} - B_g(p) i_{gp}$$

$$\dot{\theta}_p = C_g(p) e_{gp} - A_g(p) i_{gp},$$

or in matrix notation:

$$\left\| \begin{matrix} 0 \\ \dot{\theta}_{gp} \end{matrix} \right\| = \left\| \begin{matrix} D_g(p) & B_g(p) \\ C_g(p) & A_g(p) \end{matrix} \right\| \times \left\| \begin{matrix} e_{gp} \\ -i_{gp} \end{matrix} \right\|, \tag{92}$$

where

$$\begin{aligned} D_g(p) &= Z_{mg}(p)/K_g, & B_g(p) &= 1/K_g, \\ C_g(p) &= -1/K_g, & A_g(p) &= -Z_{eg}(p)/K_g. \end{aligned} \tag{93}$$

Since the output terminals of the geophone are connected directly to the input terminals of the galvanometer.

$$e_p = e_{gp}; \qquad -i_p = i_{gp}. \tag{94}$$

Consequently substituting (87) and (93) in (91),

$$\left\| \begin{matrix} 0 \\ \dot{\theta}_p \end{matrix} \right\| = \left\| \begin{matrix} D_g(p) & B_g(p) \\ C_g(p) & A_g(p) \end{matrix} \right\| \times \left\| \begin{matrix} D(p) & B(p) \\ C(p) & A(p) \end{matrix} \right\| \times \left\| \begin{matrix} f_p \\ -\dot{s}_p \end{matrix} \right\| \tag{95}$$

or

$$\left\| \begin{matrix} 0 \\ \dot{\theta}_p \end{matrix} \right\| = \left\| \begin{matrix} H_1(p) & H_2(p) \\ H_3(p) & H_4(p) \end{matrix} \right\| \times \left\| \begin{matrix} f_p \\ -\dot{s}_p \end{matrix} \right\|, \tag{96}$$

where

$$\begin{aligned} H_1(p) &= D_g(p) \times D(p) + B_g(p) \times C(p) \\ H_2(p) &= D_g(p) \times B(p) + B_g(p) \times A(p) \\ H_3(p) &= C_g(p) \times D(p) + A_g(p) \times C(p) \\ H_4(p) &= C_g(p) \times B(p) + A_g(p) \times A(p) \end{aligned} \tag{97}$$

and consequently

$$\dot{\theta}_p = f_p / \left[H_3(p) - \frac{H_4(p) \times H_1(p)}{H_2(p)} \right]. \tag{98}$$

In the expression (82) the motion of the earth has been designated by the function w and no assumptions were made as to how to represent the function w. Various attempts have been made in order to give an analytical expression of the arrival of a train of seismic waves. The most successful appears to be the one, given by P. Berlage (Reference 7).

$$w = At\epsilon^{-\beta t} \sin \omega t. \tag{99}$$

The expression (99) implies that at the initial instant of the arrival

of the train of waves the velocity of the earth motion is zero and the acceleration increases discontinuously. In order to eliminate this restriction and to express the phase relationship between the term $\sin \omega t$ and the envelope term $At\epsilon^{-\beta t}$ in a more general form it has been found preferable to adopt the function

$$w = At^m\epsilon^{-\beta t} \cos (\omega t + \phi) \tag{100}$$

to which corresponds the image function

$$w_p = A(-1)^m \frac{d^m}{d\beta^m} \frac{p(p+\beta)\cos\phi - \omega p \sin\phi}{(p+\beta)^2 + \omega^2}. \tag{101}$$

In actual cases the ground motion resulting from a dynamite explosion can be conceived as resulting from the superposition of various wave trains which originate at the point of explosion and arrive at the geophone by following certain unique paths, determined by the physical properties and certain geometrical relationships involved. Thus the actual ground motion is expressed by the image function

$$w_p = \sum_{k=1,2,\cdots} A_k\epsilon^{-\lambda_k p}(-1)^{m_k} \frac{d^{m_k}}{d\beta^{m_k}} \frac{p(p+\beta_k)\cos\phi_k - \omega_k p \sin\phi_k}{(p+\beta_k)^2 + \omega_k^2}. \tag{102}$$

In this expression the term $\epsilon^{-\lambda_k p}$ expresses the "shifting" of various arrivals along the time axis.

The corresponding motion of the galvanometer is then expressed by the image function (98) in which the value of w_p is given by (101). The corresponding time function expressing the actual motion of the galvanometer can be found by solving an integral equation similar to (81) i.e. by expressing it in a form in which θ_g will appear explicitly. This can be done by means of Fouriers infinite integral identity and the solution is expressed in form of an infinite integral in the complex p plane.

$$\theta_g = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \frac{\theta_{gp}}{p} \epsilon^{pt} dp \tag{103}$$

where $c = |c|$.

The common methods for computing this integral make use of the Expansion Theorem or the development in convergent or asymptotic series.

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EQUIVALENT ELECTRICAL NETWORKS OF SOME SEISMOGRAPHS*

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ABSTRACT

The magnetic and electrostatic geophones are represented by means of "equivalent electrical circuits" which consist of linear networks, the characteristics of which are such as to give an external behaviour identical to that of a geophone.

Various types of seismographs are considered from the standpoint of equivalent electrical networks and a few typical cases are treated quantitatively.

INTRODUCTION

In the study of dynamical systems involving mechanical elements interlinked with electrical elements a simplification may be introduced by substituting the mechanical elements by "equivalent" electrical elements. It is obvious that the "equivalency" is purely formal and results only from the similarity of the differential equations which describe the mechanical and the corresponding electrical systems.

Quoting Jeans (Reference 1):

"Imagine that we have a complicated machine in a closed room, the only connection between this machine and the exterior of the room being by means of a number of ropes which hang through holes in the floor into the room beneath. A man who cannot get into the room which contains the machine will have no opportunity of actually inspecting the mechanism, but he can manipulate it to a certain extent by pulling the different ropes. If on pulling one rope, he finds that others are set into motion, he will understand that the ropes must be connected by some kind of mechanism . . . "

The purpose of an equivalent mechanism is the substituting of a usually simpler structure in the place of the actual mechanism being considered. The equivalent structure being designed in such a manner that "the man who cannot get into the room" will not be able to see that there is a difference in the structures of the actual and equivalent mechanisms.

It is of course obvious that the matter, as to which mechanism presents a "simpler structure," can be questioned. Electrical equivalents may appear to be considerably simpler to a telephone engineer,

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but may be much harder to visualize by a steam engine specialist. However, the simplicity of electrical structures is based upon the fact that considerable work has been done in the theory of electrical networks. The results obtained can easily be extended to mechanical systems in cases in which the process of substituting the mechanical elements by the "equivalent" electrical elements is possible.

(1) MAGNETIC GEOPHONE

In the previous paper the conditions of dynamical equilibrium of a magnetic geophone were expressed as follows:

$$Z_m(p)\dot{s}_p - K_{gm}i_p = m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p \qquad (1)$$

$$Z_e(p)i_p + K_{gm}\dot{s}_p = e_p \qquad (2)$$

or in matrix notation:

$$\left\| \begin{matrix} Z_m(p) & -K_{gm} \\ K_{gm} & Z_e(p) \end{matrix} \right\| \times \left\| \begin{matrix} \dot{s}_p \\ i_p \end{matrix} \right\| = \left\| \begin{matrix} m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p \\ e_p \end{matrix} \right\| \qquad (3)$$

where:

$Z_m(p) = m_m p + r_m + 1/pC_m$ expresses the mechanical impedance,

$Z_e(p) = Lp + R$ expresses the electrical impedance,

K_{gm} is the transducing resistance,

$\dot{s}_p$ is the image function of the relative velocity $\dot{s}$ in the mechanical system,

i_p is the image function of the current i in the electrical system,

w_p is the image function of $w(t)$ expressing the earth motion,

$$w_0 = w(0) \qquad \text{and} \qquad \left(\frac{dw}{dt}\right)_0 = \left(\frac{dw}{dt}\right)_{t=0}$$

e_p is the image function of the voltage e across the output terminals of the geophone

The expressions (1), (2), or (3) represent two dynamical systems having impedances $Z_m(p)$ and $Z_e(p)$ and an asymetrical coupling determined by the transducing resistance K_{gm}. Such a coupling is different from the conventional resistance coupling and is ordinarily not encountered in electric circuit theory.

The commonly encountered resistive coupling of two electrical

meshes having impedances $Z_1(p)$ and $Z_2(p)$ is usually expressed by equations:

$$Z_1(p)i_1 - Ki_2 = e_1 \tag{4}$$

$$Z_2(p)i_2 - Ki_1 = e_2 \tag{5}$$

or in matrix notation:

$$\begin{Vmatrix} Z_1(p) & -K \\ -K & Z_2(p) \end{Vmatrix} \times \begin{Vmatrix} i_1 \\ i_2 \end{Vmatrix} = \begin{Vmatrix} e_1 \\ e_2 \end{Vmatrix} \tag{6}$$

where i_1, i_2 and e_1, e_2 denote currents and electromotive forces in the two meshes and K designates the mutual resistance.

By comparing (6) with (3) it is seen that in the matrix (6) the coupling term K is of the same sign in the upper and the lower row while the transducing resistance coupling K_{gm} has opposite signs in the two rows of matrix (3).

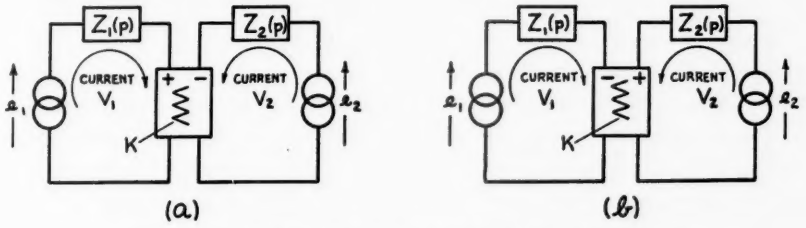

FIG. 1. Symbolic representation of a transducing resistance.

The transducing resistance coupling shall be represented symbolically as shown in Fig. 1. Figure 1-a shows a transducing resistance which would correspond to a matrix relation:

$$\begin{Vmatrix} Z_1(p) & K \\ -K & Z_2(p) \end{Vmatrix} \times \begin{Vmatrix} i_1 \\ i_2 \end{Vmatrix} = \begin{Vmatrix} e_1 \\ e_2 \end{Vmatrix} \tag{7}$$

Here the transducing resistance shall be considered as "positive" with respect to the e.m.f. e_1.

Figure 1-b corresponds to a relation:

$$\begin{Vmatrix} Z_1(p) & -K \\ K & Z_2(p) \end{Vmatrix} \times \begin{Vmatrix} i_1 \\ i_2 \end{Vmatrix} = \begin{Vmatrix} e_1 \\ e_2 \end{Vmatrix}. \tag{8}$$

Here the transducing resistance shall be designated as "negative" with respect to e_1.

Figure 2 shows an electric circuit which can be made to behave as the one shown in Fig. 1-a. The transducing resistance has been

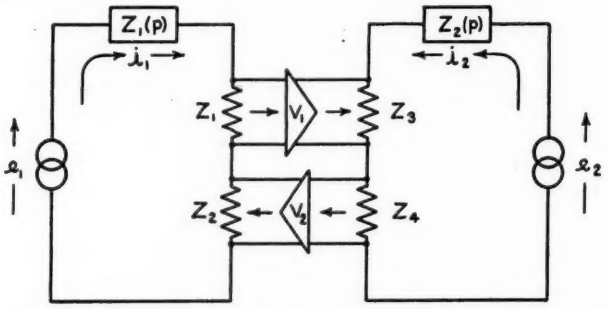

FIG. 2. Functional representation of a transducing resistance.

substituted by two electronic amplifiers V_1 and V_2 and the arrows indicate the direction of the energy transfer. The impedances Z_1, Z_2, Z_3, Z_4 are small and are made to be negligible comparing with other impedances of the circuit so that their effect can be neglected. The output voltage of the amplifier V_1 is equal to $-Ki_1$ and that of V_2 is equal to $+Ki_2$.

Assume now that there exists a box having four terminals; two of which are designated as input and two as output terminals. The box itself can be thought of as containing a linear network of some kind whose characteristics are such as to give rise to the external behaviour discussed in connection with the relations (1), (2), or (3). In order to show more explicitly the properties of this box the linear network can be represented as shown in Fig. 3; it comprises impedances $Z_m(p)$, $Z_e(p)$ and a transducing resistance K_{gm}. If an e.m.f.

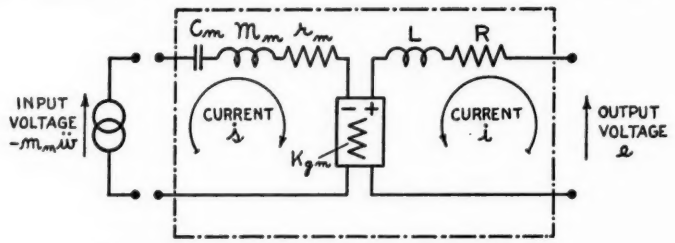

FIG. 3. Equivalent circuit of a magnetic geophone.

$-m_m\ddot{w}$ be applied across the input terminals of the box, the current and voltage delivered by the output will be always the same (under

any external load condition) as the output of the corresponding geophone when subjected to the earth motion w. At the same time current flowing into the input terminals of the box represents the instantaneous relative velocity $\dot{s}$ between the geophone case and the magnets.

Such a representation has been designated as "an equivalent circuit of a geophone." The term "equivalent circuit" does not have here perhaps its conventional meaning since it includes a transducing resistance which is not one of the conventional circuit elements.

Conventional electrical circuit elements have a definite mathematical significance and usually appear as coefficients of a differential equation. Physically they define a distinct activity in a part of an electric circuit and unfortunately are too often associated with a coil, condenser, or resistor. While this interpretation is correct, it limits, however, the scope of the meanings involved and it is customary to consider only positive capacitance or inductance or resistance. It should be observed however, that various properties of linear networks refer also to those containing the "unconventional" parameter, as the above referred to geophone circuit.

The output terminals of the geophone are connected to a recording equipment and intermediate equipment which may be considered as a dynamical system with several degrees of freedom having an operational impedance $Z(p)$. Under these conditions the output current of the geophone may be determined by substituting in (1), (2), or in (3).

$$e_p = - Z(p) i_p. \tag{9}$$

Then

$$i_p = \frac{m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p}{K_{gm} + \frac{Z_m(p) \times [Z_e(p) + Z(p)]}{K_{gm}}}. \tag{10}$$

In Fig. 4 is shown another equivalent geophone circuit which represents probably more nearly the actual physical picture. The impressed ground motion is represented here by the current $\dot{w}$. The generator supplying the current $\dot{w}$ is of infinite power and possesses an infinite internal resistance. There exists an obvious analogy between the generator supplying infinite power, a small fraction of which is absorbed by the equivalent geophone circuit, and the earth motion

the energy of which is practically unlimited comparing with the energy absorbed by the geophone. In both cases the energy flow is in one direction only, as any retroactive action of the equivalent circuit upon the generator or of the geophone upon the earth motion is negligible.

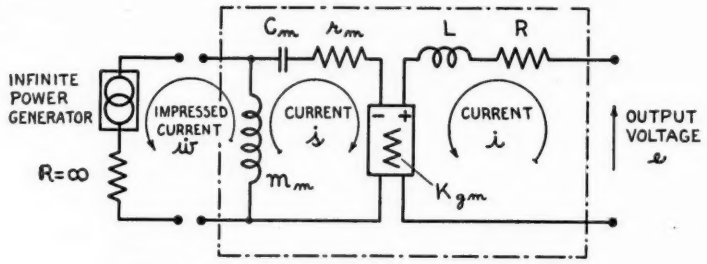

FIG. 4. Equivalent circuit of a magnetic geophone.

It can easily be shown that Fig. 4 represents the relationships expressed by the geophone equations (1), (2). Namely, the current flowing through the inductance m_m is $(\dot{w}+\dot{s})$ and the voltage across the terminals of this inductance is $-m_m(\ddot{w}+s)$. Since this voltage is applied to the circuit containing C_m in series with r_m and K_{gm}, the resulting equation is as follows:

$$-m_m(\ddot{w}+\ddot{s}) = r_m\dot{s} + \frac{1}{C_m}s - K_{gm}i. \qquad (11)$$

This equation is analogous to the equation (1). Also the relationships in the other mesh in Fig. 4 (containing L and R) are obviously expressed by the equation analogous to (2). Consequently Fig. 4 represents an equivalent geophone circuit.

An equivalent geophone circuit comprising only the conventional circuit elements is shown in Fig. 5. It consists of a four terminal

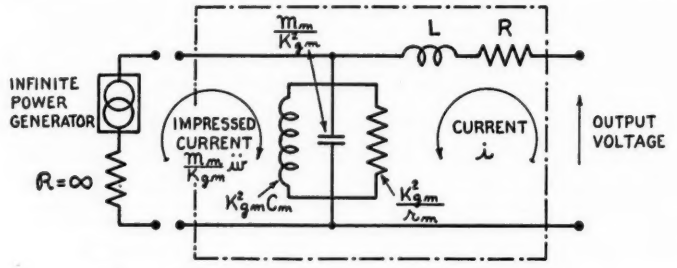

FIG. 5. Equivalent circuit of a magnetic geophone.

transducer, the input terminals of which are shunted by an inductance $K_{gm}^2 C_m$, capacitance m_m/K_{gm}^2, and resistance K_{gm}^2/r_m and one of the output terminals is connected to the resistance R in series with the inductance L. If across the input terminals be impressed a current $m_m\ddot{w}/K_{mg}$ then the voltage and current in the output will be the same as in geophone.

The equivalency between the circuit in Fig. 5 and Fig. 3 can be determined as follows: Assume that an impedance $Z(p)$ be inserted across the output terminals of the circuit in Fig. 5. Then the total admittance of the circuit as viewed from the input terminals is:

$$A(p) = \frac{1}{Z_e(p) + Z(p)} + \frac{1}{K_{gm}^2} Z_m(p) \tag{12}$$

and the admittance of the branch containing the current i is:

$$A_1(p) = \frac{1}{Z_e(p) + Z(p)}. \tag{13}$$

The impressed current $m_m\ddot{w}/K_{gm}$ can be represented by its image function.

$$\left(\frac{m_m}{K_{gm}}\ddot{w}\right)_p = \frac{m_m}{K_{gm}}\left[-\left(\frac{dw}{dt}\right)_0 - pw_0 + p^2 w_p\right]. \tag{14}$$

The ratio of the impressed current to the output current is equal to the ratio of the admittances $A(p)$ and $A_1(p)$.

$$-\frac{(m_m\ddot{w}/K_{gm})_p}{i_p} = \frac{A(p)}{A_1(p)} \tag{15}$$

whence:

$$i_p = \frac{m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p}{K_{gm} + \frac{Z_m(p) \times [Z_e(p) + Z(p)]}{K_{gm}}}. \tag{16}$$

The expression thus obtained is identical to (10). Consequently the circuit of Fig. 5 represents the relationships expressed by (1), (2) and is therefore an equivalent circuit of a geophone.

The direction of currents and voltages in the referred figures has been indicated by means of arrows. The convention as to the reference

direction is symmetrical. It should be observed that the reference direction for current i is the opposite of the one very often used in the study of geophones.

(2) ELECTROSTATIC GEOPHONE

Using image transformations the electrostatic geophone can be described by the following equation:

$$Z_m(p)\dot{s}_p - \frac{1}{pC_{gc}}\, i = m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p \qquad (17)$$

$$Z_e(p)i - \frac{1}{pC_{gc}}\,\dot{s} = e_p \qquad (18)$$

or in matrix notation:

$$\left\| \begin{matrix} Z_m(p) & -\frac{1}{pC_{gc}} \\ -\frac{1}{pC_{gc}} & Z_e(p) \end{matrix} \right\| \times \left\| \begin{matrix} \dot{s}_p \\ \dot{q}_p \end{matrix} \right\| = \left\| \begin{matrix} m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p \\ e_p \end{matrix} \right\| \qquad (19)$$

where:

$Z_m(p) = m_m p + r_m + \frac{1}{pC_m}$ is the mechanical impedance.

$Z_e(p) = R + \frac{1}{pC}$ is the electrical impedance.

C_{gc} is the mutual capacitance.

and $\dot{s}_p$, i_p, e_p, w_p, $\left(\frac{dw}{dt}\right)_0$, w_0 have the same meaning as in equation (1), (2).

It is noted that the matrix representing the capacity geophone is of the form:

$$\left\| \begin{matrix} Z_m(p) & -\frac{1}{pC_{gc}} \\ -\frac{1}{pC_{gc}} & Z_e(p) \end{matrix} \right\| \qquad (20)$$

and the intercoupling factor $-1/pC_{gc}$ is of the same sign in both rows. This matrix is similar to a matrix

$$\left\| \begin{matrix} Z_A(p) + \dfrac{1}{pC_{gc}} & -\dfrac{1}{pC_{gc}} \\ -\dfrac{1}{pC_{gc}} & Z_B(p) + \dfrac{1}{pC_{gc}} \end{matrix} \right\| \tag{21}$$

which represents an electrical T-structure shown in Fig. 6. In order

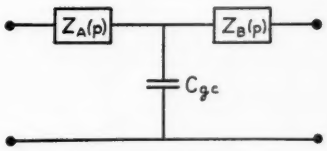

FIG. 6. T-Structure represented by matrix (21).

that this T-structure represent the equivalent circuit of the geophone, it is necessary that (21) be identical to (20) and consequently:

$$Z_m(p) = Z_A(p) + \frac{1}{pC_{gc}} \tag{22}$$

$$Z_e(p) = Z_B(p) + \frac{1}{pC_{gc}} \tag{23}$$

whence:

$$Z_A(p) = Z_m(p) - \frac{1}{pC_{gc}} = m_m p + r_m + \frac{1}{p}\left(\frac{1}{C_m} - \frac{1}{C_{gc}}\right) \tag{24}$$

$$Z_B(p) = Z_e(p) - \frac{1}{pC_{gc}} = R + \frac{1}{p}\left(\frac{1}{C} - \frac{1}{C_{gc}}\right). \tag{25}$$

If:

$$\frac{1}{C_m} - \frac{1}{C_{gc}} > 0 \tag{26}$$

$$\frac{1}{C} - \frac{1}{C_{gc}} > 0 \tag{27}$$

the equivalent circuit of the geophone is as shown in Fig. 6. If how-

ever, both or one of the relations (26), (27) are not satisfied, the equivalent circuit will contain negative capacitance. The negative capacitances, though it does not belong to the conventional electrical circuit parameters, can at least theoretically be made (Reference 3). A negative capacitance circuit is shown in Fig. 7. It includes a negative resistance $-R$ which can be made by means of a dynatron. Since dynatron circuits are not very stable it would be desirable to have a network containing only positive circuit parameters.

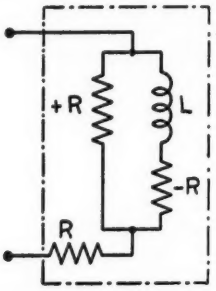

FIG. 7. Negative capacitance circuit.

Consider again the dynamical system as represented by the matrix (20). The necessary conditions for physical realizability is that the potential energy function, which is a quadratic function of s and q, should be positive for *all* values of the variable s and q. Stated mathematically this condition requires that the determinant formed by the elastance coefficients be positive (Reference 4).

$$\begin{vmatrix} \frac{1}{C_m} & -\frac{1}{C_{gc}} \\ -\frac{1}{C_{gc}} & \frac{1}{C} \end{vmatrix} \geqq 0 \tag{28}$$

or

$$\frac{1}{CC_m} + \frac{1}{C_{gc}^2} \geqq 0. \tag{29}$$

This condition is always satisfied. Assume now that the conditions (26) and (27) do not hold and consequently the equivalent geophone circuit cannot be represented as shown in Fig. 6. Take for instance a particular example in which:

$$\frac{1}{C_m} - \frac{1}{C_{ge}} < 0 \tag{30}$$

$$\frac{1}{C} - \frac{1}{C_{ge}} < 0. \tag{31}$$

Under the conditions as expressed by (30), (31) the matrix (20) can be made to correspond to a dissipative network by insertion of an ideal transformer, i.e., a transformer which neither stores nor dissipates energy and in which there is a perfect flux linkage between the windings. This can be shown as follows: Consider now a four terminal network represented by the matrix

$$\left\| \begin{array}{cc} Z_{11}(p) & -\frac{1}{pC_{12}} \\ -\frac{1}{pC_{21}} & Z_{22}(p) \end{array} \right\|. \tag{32}$$

If this network be preceded by an ideal transformer having a ratio $a:1$ then the corresponding matrix can be written as:

$$\left\| \begin{array}{cc} a^2 Z_{11}(p) & -a\frac{1}{pC_{12}} \\ -a\frac{1}{pC_{12}} & Z_{22}(p) \end{array} \right\|. \tag{33}$$

Assume now that the network represented by the matrix (33) is the equivalent network of the geophone and consequently the matrices (33) and (20) are identical.
Then:

$$Z_m(p) = a^2 Z_{11}(p) = a^2\left[pL_{11} + R_{11} + \frac{1}{pC_{11}}\right] \tag{34}$$

$$\frac{a}{C_{12}} = \frac{a}{C_{21}} = \frac{1}{C_{ge}} \tag{35}$$

$$Z_e(p) = Z_{22}(p). \tag{36}$$

In order to avoid the negative capacitance it is necessary that:

$$\frac{1}{C_{11}} - \frac{1}{C_{12}} > 0 \tag{37}$$

or

$$a^2C_m - aC_{gc} < 0. \tag{38}$$

Thus:

$$a < \frac{C_{gc}}{C_m}. \tag{39}$$

Thus the parameters of the equivalent geophone circuits consists of the values a, $Z_{11}(p)$, $Z_{22}(p)$, C_{12} and are determined by the equations (34), (35), (36), and (39). The equivalent geophone circuit is shown in Fig. 8.

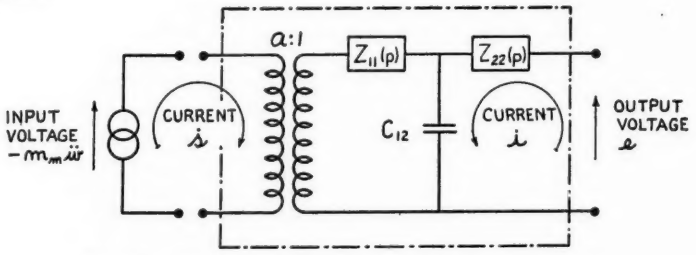

FIG. 8. Equivalent circuit of an electrostatic geophone.

Assume now that the output terminals of the geophone are connected to a recording and intermediate equipment having an operational impedance $Z(p)$. Then

$$e_p = -Z(p)i_p$$

and substituting this expression in (17) and (18) the output current of the geophone can be expressed as follows:

$$i_p = \frac{\left[m_m\left(\frac{dw}{dt}\right)_0 + m_m w_0 - m_m p^2 w_p\right]/pC_{gc}}{[Z_m(p) \times Z_e(p) + 1/p^2C_{gc}^2]}. \tag{40}$$

GALVANOMETER

The conditions of dynamical equilibrium in a galvanometer have been expressed as follows:

$$Z_{eg}(p)i_{gp} + K_g\dot{\theta}_{gp} = e_{gp} \tag{41}$$

$$Z_{mg}(p)\dot{\theta}_{gp} - K_g i_{gp} = 0 \tag{42}$$

or in matrix notation:

$$\begin{Vmatrix} Z_{eg}(p) & K_g \\ -K_g & Z_{mg}(p) \end{Vmatrix} \times \begin{Vmatrix} i_{gp} \\ \dot{\theta}_{gp} \end{Vmatrix} = \begin{Vmatrix} e_{gp} \\ 0 \end{Vmatrix} \tag{43}$$

where:

$Z_{eg}=L_{gp}+R_g$ is the electrical impedance,

$Z_{mg}=I_{mg}p+r_{mg}+\frac{1}{pC_{mg}}$ is the mechanical impedance,

K_g is the transducing resistance,

i_{gp} is the image function of the current in the galvanometer coil

θ_{gp} is the image function of the angular deflection θ_{gp} of the galvanometer and

e_{gp} is the image function of the e.m.f. e_g impressed upon the galvanometer.

The relations (41), (42), or (43) can be expressed also by means of an equivalent galvanometer circuit shown in Fig. 9. The equivalent circuit consists of two meshes possessing impedances $Z_{eg}(p)$, $Z_{mg}(p)$ and intercoupled by a transducing resistance K_g. The e.m.f. e_g is applied to the first mesh and the current in this mesh represents the actual current flowing through the galvanometer coil. The electrical displacement in the second mesh represents the instantaneous value of the galvanometer deflection.

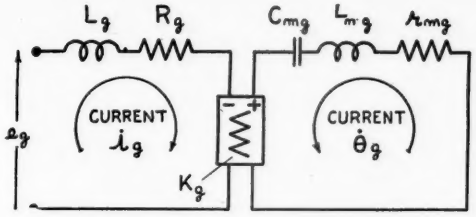

FIG. 9. Equivalent circuit of a galvanometer.

SEISMOGRAPH

A seismograph has been defined as "an arrangement for detecting and recording earth vibrations." It includes usually: a geophone, oscillograph, amplifiers, and often an additional element designated here as "intermediate network."

Figure 10 shows a seismograph used for geophysical prospecting. It consists of six small, completely encased, moving armature type

geophones and of a recording station mounted on a truck which contains the amplifying and the registering instruments. All the apparatus are ruggedly constructed in order to withstand the necessary handling and transportation in the field.

The geophones and oscillograph have been briefly described in the previous paper. The amplifiers are of special design so as to magnify the impressed transient without any distortion. An amplifier unit is shown in Fig. 11.

One of the constituents of a seismograph, designated above as

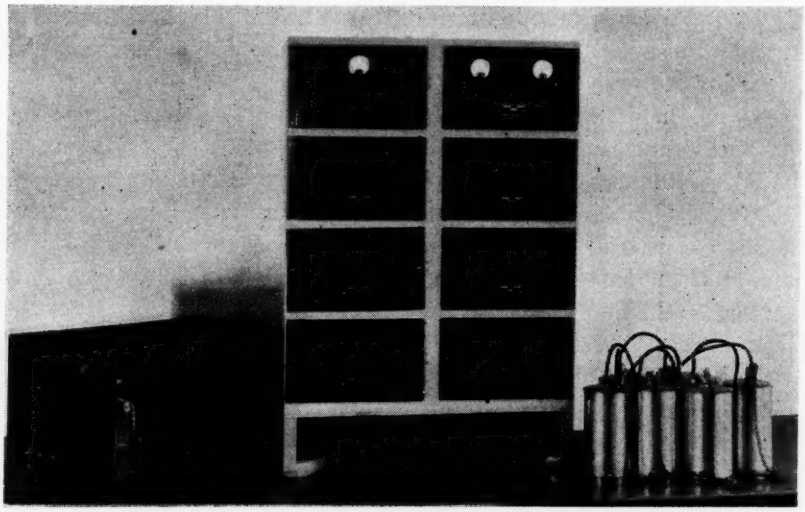

FIG. 10. Seismograph.

"intermediate network" consists of a four terminal transducer inserted between the geophone and the oscillograph. Since the present discussion refers to structural characteristics of a seismograph rather than to its functional properties, only a very few remarks on the "intermediate network" are given here and a more detailed discussion will be contained in a subsequent paper. In general, the functional properties of an "intermediate network" in earthquake recording are entirely different from those in seismic reflection prospecting. In earthquake recording the "intermediate network" is essentially a distortion correcting device while in the geophysical seismograph it is a distortion producing device.

From the standpoint of ideal quality, an earthquake seismograph

must be so designed that the recorded motion of the galvanometer which represents the earth vibrations shall be a faithful copy of the impressed earth motion; that is the seismograph must be distortionless.

In order to determine the departure of the recorded motion from the true ground motion it is convenient to define the properties of the seismograph in terms of changes that occur in the amplitude and the phase of steady state sinusoidal earth waves of different frequencies.

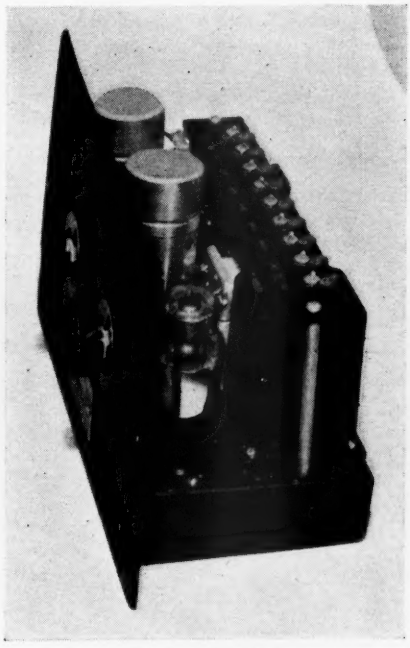

FIG. 11. Amplifier.

Thus the terms: attenuation characteristic and phase characteristics are introduced which define the amplitude change and the phase change as a function of frequency.

For an ideal quality earthquake seismograph, the attenuation should be independent of frequency and the phase should be proportional to frequency. In an actual seismograph the problem of distortion correction is that of altering the arrangement in some way so as to approach this ideal; this can be done by interposing between the geophone and the recording galvanometer a transducer having the requisite properties. Thus the term "intermediate network" in the

case of an earthquake seismograph refers to an arrangement for correcting distortion.

The functional properties of the "intermediate network" used in a geophysical reflection seismograph are different. It is not desired here to record distortionlessly the seismic motion—but to indicate clearly the instant of the arrival of a reflected wave. To accomplish this purpose certain distortion is introduced willfully by impressing the geo-

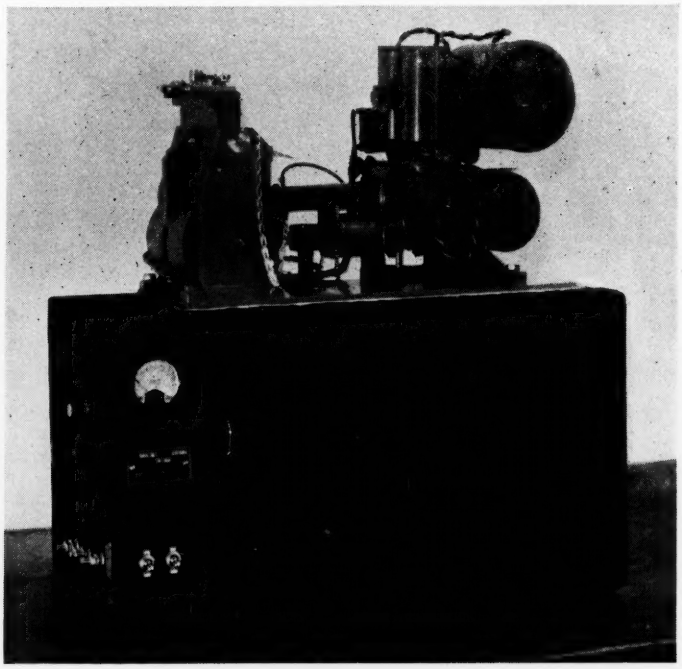

FIG. 12. Oscillograph.

phone output voltage upon the "intermediate network." This "intermediate network" has a structure resembling an electric wave filter.

In the earlier stages of the geophysical seismograph main emphasis was put on the steady state characteristics of the "filter" with a view towards eliminating "ground roll" (i.e., low frequency surface wave effects). This in itself can be very easily accomplished and any desired degree of "ground roll" suppression realized. The emphasizing of the arrival instant of a reflected wave has been recognized only more recently and it is this function that constitutes the main object of the present "intermediate networks."

An "intermediate network" assembly is shown in Fig. 13.

Throughout the present paper the term "intermediate network" shall be continually used though it is understood to designate a corrective network in earthquake seismology and a distorting network in connection with a geophysical seismograph.

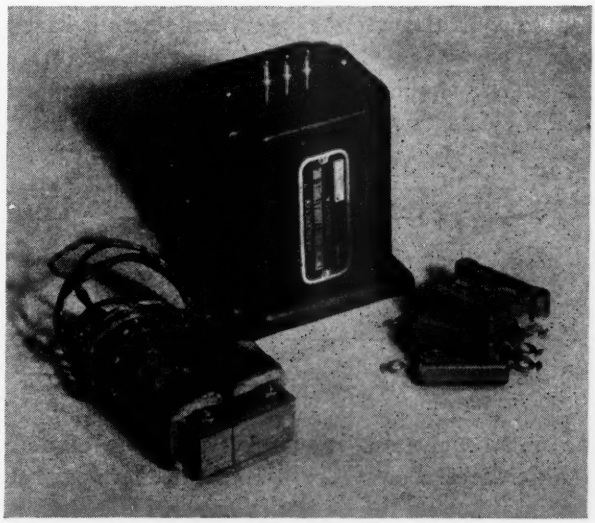

FIG. 13. Electric filter.

There exists a great variety of seismograph types. In order to facilitate further discussion six typical arrangements shall be considered and identified as seismograph type *A*, *B*, *C*, *D*, *E* and *F*. The seismograph type *A* consists of an arrangement: "Geophone-Oscillograph," type *B* of "Geophone-Intermediate Network-Oscillograph," type *C* of "Geophone-Amplifier-Oscillograph," type *D* of "Geophone-Intermediate Network-Amplifier-Oscillograph," type *E* of "Geophone-Amplifier-Intermediate Network-Oscillograph," type *F* of "Geophone-Amplifier-Intermediate Network-Amplifier-Oscillograph" A schematic representation of the amplifier type *F* is shown in Fig. 14.

FIG. 14. Seismograph Type F.

Consider now seismograph type B. Each element of the seismo-

graph can be represented by an "equivalent" electrical circuit, as is shown in Fig. 15. This "equivalent" of a seismograph consists of a number of meshes mutually coupled one to another and which represent the various seismograph elements. It has been assumed here that the "intermediate network" possesses a ladder structure consisting of impedances.

Usually the equivalency of electrical structures is based upon the similarity in their external behaviour. That is to say, if the actual mechanism and its equivalent were enclosed in separate boxes, the only accessible elements of which were the input and the output ter-

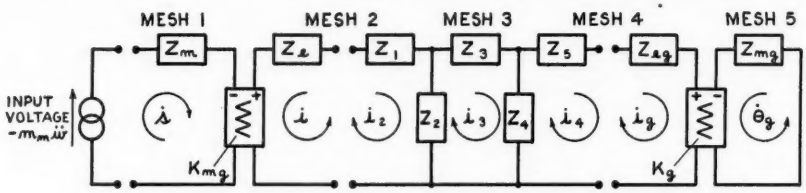

FIG. 15. Equivalent network of a seismograph Type B.

minals, an outside observer would be incapable of detecting any structural differences. However, in the case of Fig. 15 the equivalency is based essentially upon the similarity between the internal structure of the seismograph and its representative network. Here, in the equivalent network the mechanical elements have been substituted by electrical, the mechanical relationships have been translated into electrical. Thus in the mesh (1) the input voltage $-m_m\ddot{w}$ represents the earth acceleration, the current $\dot{s}$ corresponds to the relative instantaneous velocity between the geophone casing and magnets; in the mesh (2) flows the actual current delivered by the geophone; the meshes (2, 3, 4) include the intermediate network; in the mesh (4) flows the actual current flowing through the galvanometer coil; and the electrical displacement in the mesh (5) represents the deflection θ_g of the galvanometer.

In some instances it may be desired to represent the seismograph in the form of an electrical circuit consisting of only one mesh in which the relationships would be more explicitly stated. Consider for instance Fig. 16. It represents an electrical circuit consisting only of one mesh. The input voltage $-m_m\ddot{w}$ corresponds to the earth acceleration, the electric displacement represents the galvanometer deflection and the impedance is a function of p which depends upon all the param-

eters of the seismograph constitutents and expresses among other things the contribution which is due to the reaction of the meshes (1, 2, 3, 4) upon the mesh (5) in Fig. 15. The circuit as represented here is fictitious; it is however useful in order to visualize the relationship between one particular mesh, the impressed force and the reaction of the remaining meshes.

In a similar manner the equivalent circuit shown in Fig. 17 represents the relation between the impressed earth motion and the motion of the geophone.

The networks of Fig. 16 and Fig. 17 shall be designated as "representing the galvanometer system" and "representing the geophone system" in a seismograph.

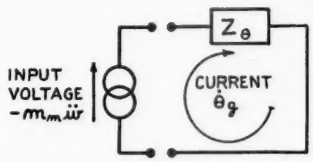

FIG. 16. Circuit representing the mechanical system of the galvanometer.

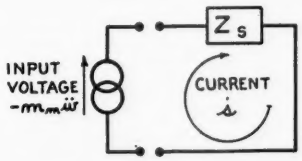

FIG. 17. Circuit representing the mechanical system of the geophone.

The networks of Fig. 16 and 17 which represent the behaviour of one of the component parts of a seismograph can be derived from the equivalent network of Fig. 15 by means of a certain sequence of operations based on the inversion of electrical networks, on Bartlett's Reciprocation Theorem, and on some properties of the transducing resistance.

INVERSE NETWORKS

If given three networks Z_a, Z_b, Z_c where $Z_a \equiv Z_a(p)$ etc. and the relation

$$Z_a Z_b = Z_c^2 \tag{44}$$

the network Z_a is considered as inverse of Z_b with respect to Z_c. If Z_c is real, Z_a is a reciprocal of Z_b.

BARTLETT'S RECIPROCATION THEOREM (Reference 5)

Consider a network of a ladder structure shown in Fig. 18. The

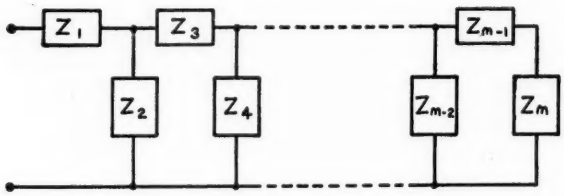

FIG. 18. Two terminal ladder structure.

driving point impedance of this network is expressed by a continued fraction:

$$Z_1 + \cfrac{1}{\cfrac{1}{Z_2^{-1}} + \cfrac{1}{\cfrac{1}{Z_3} + \cfrac{1}{\cfrac{1}{Z_4^{-1}} + \cdots + \cfrac{1}{\cfrac{1}{Z_{n-1}} + \cfrac{1}{\cfrac{1}{Z_n^{-1}}}}}}} \tag{45}$$

and the driving point impedance of a reciprocal network with respect to a constant K^2 is expressed as follows:

$$\cfrac{1}{\cfrac{Z_1}{K^2}} + \cfrac{1}{\cfrac{K^2}{Z_2}} + \cfrac{1}{\cfrac{Z_3}{K^2}} + \cfrac{1}{\cfrac{K^2}{Z_4}} + \cdots + \cfrac{1}{\cfrac{Z_{n-1}}{K^2}} + \cfrac{1}{\cfrac{K^2}{Z_n}} \tag{46}$$

It is readily seen, however, that the expression (46) represents the network shown in Fig. 19. Consequently the network of Fig. 19 is a reciprocal of the network in Fig. 18.

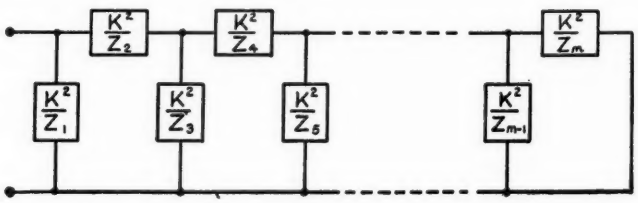

FIG. 19. Reciprocal of the structure of Fig. 23.

TRANSDUCING RESISTANCE

Consider two meshes having impedances Z_a, Z_b and intercoupled by means of a transducing resistance K. Let an electromotive force be applied to the mesh (a) (Fig. 20).

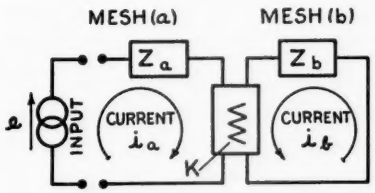

FIG. 20. Two terminal network.

The equations are as follows:

$$\left\| \begin{matrix} Z_a & K \\ -K & Z_b \end{matrix} \right\| \times \left\| \begin{matrix} i_a \\ i_b \end{matrix} \right\| = \left\| \begin{matrix} e \\ o \end{matrix} \right\| \tag{47}$$

if the transducing resistance is positive with respect to the applied e.m.f. and

$$\left\| \begin{matrix} Z_a & -K \\ K & Z_b \end{matrix} \right\| \times \left\| \begin{matrix} i_a \\ i_b \end{matrix} \right\| = \left\| \begin{matrix} e \\ o \end{matrix} \right\| \tag{48}$$

if the transducing resistance is negative with respect to the applied e.m.f.

From the equations (47):

$$i_a = e/\left(Z_a - \frac{K^2}{Z_b}\right) \tag{49}$$

and

$$i_b = e/\left(\frac{Z_a Z_b}{K} + K\right). \tag{50}$$

From the equations (48):

$$i_a = e/\left(Z_a + \frac{K^2}{Z_b}\right) \tag{51}$$

and

$$i_b = -e/\left(\frac{Z_a Z_b}{K} + K\right). \tag{52}$$

From the above the following two rules may be established.

RULE 1

The driving point impedance consists of Z_a in series with the reciprocal of Z_b with respect to K^2 and can be expressed as Z_a+K^2/Z_b (Fig. 21). This rule is independent of the sign of transducing resistance with respect to the applied e.m.f.

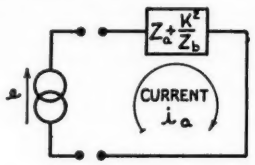

FIG. 21. Circuit resulting from the driving point transformation.

RULE 2

The transfer impedance between the input terminals and the mesh (b) consists of resistance K in series with the impedance Z_aZ_b/K and can be expressed as $K+Z_aZ_b/K$ (Fig. 22). In this rule the conventional reference direction of the current in the mesh (b) should be reversed if the transducing resistance is negative with respect to the applied e.m.f. This can be easily remembered by assuming that Z_a and Z_b are positive quantities. If the transducing resistance is positive with respect to the e.m.f., K can also be assumed as being positive. In this case the transfer impedance $K+Z_aZ_b/K$ is positive and the conventional reference direction is maintained. If, however, the transducing resistance is negative with respect to the applied e.m.f., K can be assumed as being negative and consequently the sign of the transfer impedance $K+Z_aZ_b/K$ is reversed. Then the conventional reference direction should also be reversed.

The transformation from Fig. 20 to Fig. 21 will be designated as "driving point transformation" and the transformation from Fig. 20 to Fig. 22 as "transfer transformation."

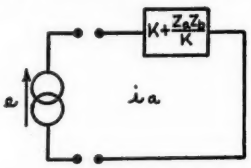

FIG. 22. Circuit resulting from the transfer transformation.

As an example consider the transformations which lead from the equivalent seismograph network of Fig. 15 to a circuit representing the motion of the geophone (Fig. 17). These are shown in Fig. 23 and

Fig. 24. Figure 23 represents the driving point transformation applied to the galvanometer system and Fig. 24 shows the further step resulting from the driving point transformation applied to the geophone system and the application of Bartlett's Reciprocation Theorem. Thus Fig. 24 shows more explicitly the structure of the circuit in Fig. 17. The applied e.m.f. $-m\ddot{w}$ represents the earth motion, the input current represents the instantaneous relative velocity in the mechanical system of the geophone, and the circuit shows how the parameters of the seismograph affect the motion of the geophone.

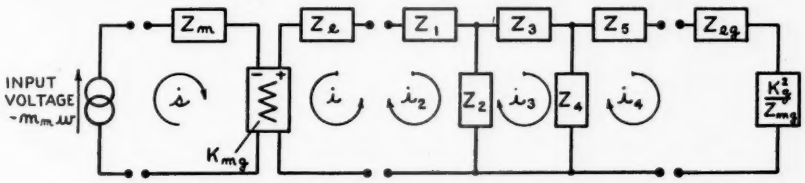

FIG. 23. A transformation of the network of Fig. 20.

Thus by means of Fig. (24) it is possible to determine analytically the geophone motion in which all the parameters of the system will be

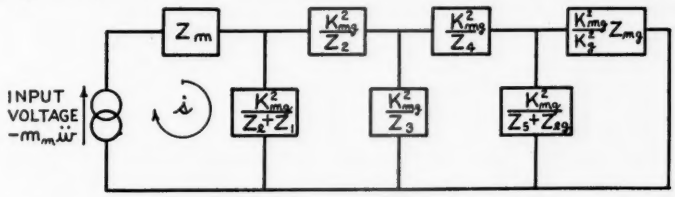

FIG. 24. Circuit representing the mechanical system of the geophone.

taken into account. Assume a suddenly impressed earth motion w. Then from Fig. 24.

$$\dot{s}_p = \frac{m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p}{Z_1} \tag{53}$$

where Z_1 is directly determined

$$Z_1 = Z_m + \cfrac{1}{\cfrac{Z_e + Z_1}{K_{mg}^2}} + \cfrac{1}{\cfrac{K_{mg}^2}{Z_2}} \quad \cfrac{1}{\cfrac{Z_3}{K_{mg}^2}} + \cfrac{1}{\cfrac{K_{mg}^2}{Z_4}} + \cfrac{1}{\cfrac{Z_5 + Z_{eg}}{K}} + \cfrac{1}{\cfrac{K\,\frac{g}{m}}{Z_{mg}K_g^2}}. \tag{54}$$

Consequently the function s is determined from its image by means of the transformation

$$s = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{s_p}{p}\epsilon^{pt}dp. \tag{55}$$

where $c=|c|$.

Figures 25, 26 show another example in which the equivalent seismograph network is transformed into a circuit representing the motion of the galvanometer (as shown in Fig. 16). Here Fig. 26 shows

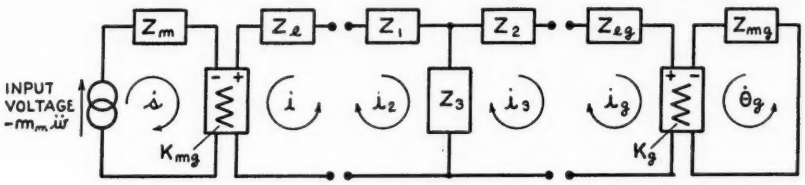

FIG. 25. Equivalent network of a seismograph Type B.

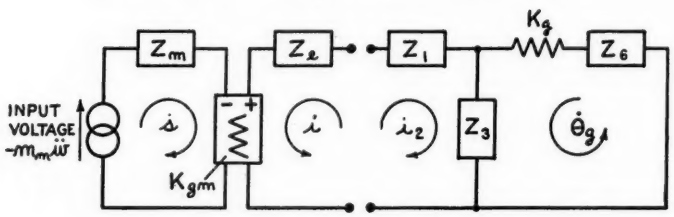

FIG. 26. A transformation of Fig. 30.

the result of applying the transfer transformation to the galvanometer and consequently the impedance Z_6 in the galvanometer circuit can be expressed as follows:

$$Z_6 = \frac{(Z_2 + Z_{eg})Z_{mg}}{K_g} \tag{56}$$

and the reference direction of $\dot{\theta}_g$ remains the same. A further transformation may be effected in such a manner that the circuit into which the geophone feeds may be shown as two impedances in series: Z_1 and Z_7. The impedance Z_7 can be expressed as follows:

$$Z_7 = \frac{Z_3(K_g + Z_6)}{Z_3 + K_g + Z_6}. \tag{57}$$

A further transfer transformation will reduce Fig. 26 to the form

represented in Fig. 16, wherein

$$Z_\theta = K_{gm} + \frac{Z_m(Z_e + Z_1 + Z_7)}{K_{gm}} \tag{58}$$

and the reference sign for $\dot{\theta}_g$ is reversed. This circuit represents the mechanical system of the galvanometer and shows how the motion of the galvanometer is affected by the impressed earth motion and by all the parameters of all the constituents of the seismograph.

If the impressed earth motion be represented as w then the motion recorded by the galvanometer will vary according to the time function the image of which is represented as:

$$\dot{\theta}_p = \frac{m_m\left(\frac{dw}{dt}\right)_0 + m_m p w_0 - m_m p^2 w_p}{K_{gm} + Z_8(p)}. \tag{59}$$

Consequently the function $\dot{\theta}$ can be expressed as follows:

$$\dot{\theta} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{\dot{\theta}_p}{p}\epsilon^{pt}dp. \tag{60}$$

where $c = |c|$.

Consider now a seismograph containing amplifiers. These provide a unidirectional coupling from the grid circuit to the plate circuit in which any retroactive influence is not possible. Take, for example, a simple arrangement as shown in Fig. 27 representing two meshes

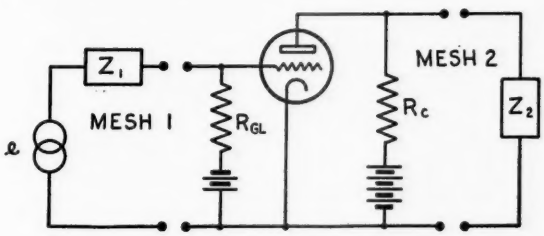

FIG. 27. Two meshes intercoupled by means of a vacuum tube.

intercoupled by means of a vacuum tube. In Fig. 28 the two meshes are separated and the conditions resulting from the presence of a vacuum tube are more explicitly shown. Thus in the mesh (1) the

vacuum tube acts as an additional impedance which is substantially equal to the grid leak resistance R_{GL} of the tube. The voltage drop across the resistance R_{GL} is:

$$e_g = \frac{eR_{GL}}{Z_1 + R_{GL}}. \tag{61}$$

In the mesh (2) the vacuum tube acts as a generator having an e.m.f. $-\mu e_g$ (μ is the coefficient of amplification of the tube) and an internal resistance equal to the plate resistance R_p of the tube.

A vacuum tube arrangement somewhat more complicated is shown in Fig. 29 and represents a seismograph type F. Here an amplifier V_1 is inserted between the geophone and the intermediate network and another amplifier V_2 is inserted between the intermediate network and

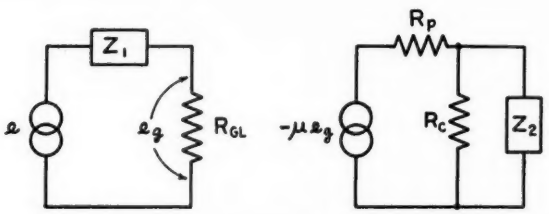

FIG. 28. A transformation of Fig. 34.

the galvanometer. Let μ_1, μ_2, R_{p1}, R_{p2}, R_{GL1}, R_{GL2}, R_{c1}, R_{c2} be the respective values of the coefficient of amplification, plate resistance, grid leak resistance, and coupling resistance of the tubes V_1 and V_2. Let also i, i, i_3, i_4, i_g and $\dot{\theta}_g$ be the currents in the successive meshes as shown in Fig. 29.

The amplifiers V_1 and V_2 divide the seismograph into three parts which are represented separately in Fig. 30. Figure 30-a represents the geophone circuit across the output terminals of which is inserred the resistance R_{GL1}. Figure 30-b represents the intermediate network possessing across its input terminals an electromotive force $e_1 = \mu_1 R_{LG1} i$ a series resistance R_{p1} and a shunt resistance R_{c1}. The output terminals of the intermediate network are connected to the resistance R_{GL2}. Figure 30-c represents the galvanometer circuit across the input terminals of which is inserted an electromotive force $e_2 = -\mu_2 R_{GL2} i_2$ in series with the resistance R_{p2} and shunted by the resistance R_{c2}.

Figure 31 (a) and (c) result from the transfer transformation of Fig. 30 (a) and (c) respectively. In Fig. 31-a the current:

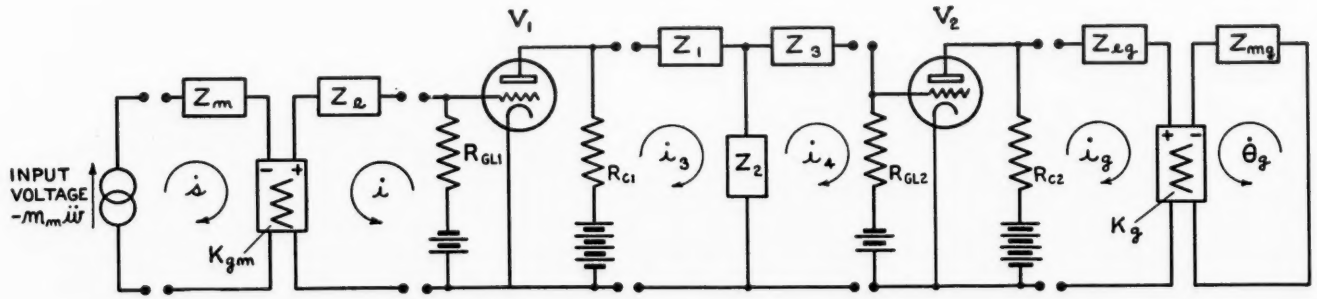

FIG. 29. Equivalent network of a seismograph type F.

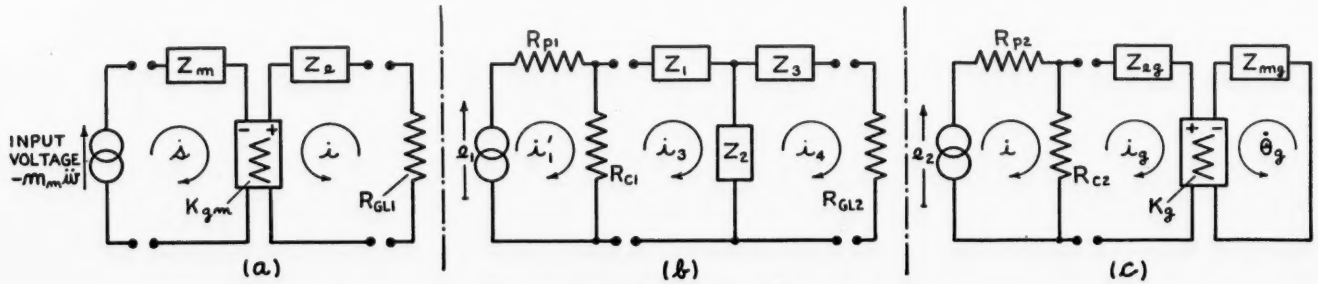

FIG. 30. A transformation of Fig. 36.

FIG. 31. A transformation of Fig. 37.

$$i_p = \frac{m_m\left(\frac{dw}{dt}\right)_0 + m_m w_0 - m_m p^2 w_p}{K_{gm} + \frac{Z_m(Z_e + R_{GL1})}{K_{gm}}}. \tag{62}$$

In Fig. 31-b the impressed voltage:

$$e_{1p} = -\mu_1 R_{GL1} i_p \tag{63}$$

and the current relationships in the various meshes can be expressed as follows:

$$\begin{aligned} (R_{p1} + R_{c1})i_p' - R_{c1}i_{3p} &= e_{1p} \\ -R_{c1}i_p' + (R_{c1} + Z_1 + Z_2)i_{3p} - Z_2 i_{4p} &= 0 \\ -Z_2 i_{3p} + (Z_2 + Z_3 + R_{GL2})i_{4p} &= 0 \end{aligned} \tag{64}$$

whence

$$i_4 = \frac{\Delta_1}{\Delta} \tag{65}$$

$$\Delta = \begin{vmatrix} (R_{p1} + R_{c1}) & -R_{c1} & 0 \\ -R_{c1} & (R_{c1} + Z_1 + Z_2) & -Z_2 \\ 0 & -Z_2 & (Z_2 + Z_3 + R_{GL2}) \end{vmatrix} \tag{66}$$

$$\Delta_1 = \begin{vmatrix} (R_{p1} + R_{c1}) & -R_{c1} & e_{1p} \\ -R_{c1} & (R_{c1} + Z_1 + Z_2) & 0 \\ 0 & -Z_2 & 0 \end{vmatrix}. \tag{67}$$

In Fig. 31-c the electromotive force

$$e_{2p} = -\mu_2 R_{GL2} i_{4p}$$

and the current

$$i_{4p}' = e_{2p}\left[\left|\frac{1}{R_{p2}}\right| + \left|\frac{1}{R_{c2}^{-1}}\right| + \left|\frac{1}{K_g}\right| + \left|\frac{1}{\left(\frac{Z_{mg}Z_{eg}}{K_g}\right)^{-1}}\right|\right]$$

Consequently:

$$\dot{\theta}_p = i_{4p}' \times \frac{\left(K_g + \frac{Z_{mg}Z_{eg}}{K_g}\right)^{-1}}{R_{c2}^{-1} + \left(K_g + \frac{Z_{mg}Z_{eg}}{K_g}\right)^{-1}}$$

and

$$\dot{\theta} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{\dot{\theta}_p}{p}\epsilon^{pt}dp$$

where $c = |c|$.

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Reference 1: J. H. Jeans, "The Mathematical Theory of Electricity and Magnetism," Third Edition, Cambridge University Press, 1915, p. 486.
Reference 2: "Fundamental Relations in Seismometry."
Reference 3: See B. van der Pol, Proc. I.R.E. 1930, p. 221.
Reference 4: E. A. Guillemin, "Communication Networks," Vol. II, John Wiley and Sons, New York, N. Y. 1935.
Reference 5: A. C. Bartlett, Phil. Mag., Vol. 49, pp. 728–730, 1925.

EXPERIMENTAL DETERMINATION OF THE TRANSIENT CHARACTERISTICS OF SEISMOGRAPH APPARATUS*

HAROLD W. WASHBURN†

ABSTRACT

A method of experimentally determining the transient response of seismometer, amplifier, and oscillograph is described. In the apparatus employed the earth motion is replaced by a transient wave generator and the seismometer by an equivalent electrical circuit.

Comparison of these laboratory data with field records has shown that accurate predictions can be made from laboratory tests.

INTRODUCTION

One of the most important problems that one confronts in reflection shooting is the separation of reflected energy from the unwanted energies of ground roll, wind disturbance, and other extraneous disturbances. In general, this separation is accomplished at three different parts of the process. That is, the relative amount of reflected energy produced at the seismometers may be increased *first* by proper field procedure, *second*, by using a series connection of seismometers, and *third*, by appropriate design of instrument characteristics. The appropriate design of instruments, may be defined as a design in which the overall characteristics are such that the instruments will emphasize the band of frequencies which is responsible for the major part of the reflected energy. Experience has shown that a certain amount of this emphasis is desirable, but that the type and amount must be carefully chosen. For example, it is possible to make the reflections on a record more distinct by proper frequency discrimination, but it is also possible to partially obliterate reflections by improper frequency discrimination. The determination of just what is the best type and amount of frequency discrimination is a very complicated problem. In this paper, it is proposed to outline an accurate and rapid experimental method of attack.

The first factor to be considered, in developing a laboratory method of studying the transient behavior of apparatus, is the form of the transient to be applied to the apparatus. We have experimented a great deal with idealized forms of reflections and also with

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combinations of idealized forms. Our experience has shown that even though the response of an apparatus to the motion resulting from a field shot is entirely determinable from its response to a single pulse, there are certain important characteristics which are extremely difficult to predict from a study of the response to a single pulse. This is due to the extremely complicated nature of the energy received by the seismometer, and to the fact that the response of the instruments depends on the energy which arrived just previous to the reflection as well as on the form of the reflection wave itself. For these reasons, in a laboratory study of the transient characteristics of apparatus, it is desirable to use a replica of the ground motion resulting from actual field shots. In order to obtain the ground motion it is necessary to use a type of apparatus which has no discrimination or distortion. It is, of course, desirable to obtain records from several localities for use as a source of transient motions.

The next factor to be decided on, is the method of applying these recorded motions to the apparatus in question. The most obvious attack is to employ a shaking table in conjunction with the proposed seismometer, amplifier and oscillograph. This method has two major drawbacks. In the first place, it is necessary to build all types of seismometers that one desires to test. Secondly, a shaking table which can be made to move in accordance with the ground motion of any desired field shot would be very costly to build; it would take considerable time to develop, and even after developed, it would in all probability be difficult to operate.[1] This type of shaking table should not be confused with the type of table which is used to test seismometers with a steady state or an idealized reflection motion. The steady state type of table is relatively simple to build. It serves well in determining the characteristics of a seismometer but is not adapted to the problem under consideration.

All of these complications of building various seismometers and developing a super shaking table can readily be avoided by replacing the seismometers with equivalent electrical circuits and replacing the shaking table with a wave generator. That is, instead of a shaking table shaking a seismometer, we have an electrical wave generator

[1] The type of shaking table referred to here has been developed by A. C. Ruge for the relatively lower frequencies encountered in earthquake motions. A description of this table is given in the Seismological Society of American Bulletin 26, pp. 201–205, 1936. The complicated nature of the structures being subjected to earthquake motions however justify the development of such a table, whereas, it is much simplier to replace the shaking table with a wave generator for seismometer testing.

feeding an equivalent electrical seismometer, the output of which is connected to the amplifier and oscillograph. (See Fig. 1.)

The wave generator must be of a type which is capable of producing a voltage wave of any desired shape. The generator which we have built for this purpose is a photo-electric type and can be made to produce any desired voltage wave by the insertion of the proper disc. These discs are accurately made from a field record of ground motion by a photographic process.

The equivalent electrical circuit which is used is not of the conventional type, but is an inverted type which was suggested in an article written by F. A. Firestone.[2] The inverted type enables one to represent acurately not only the action of the ground motion on the

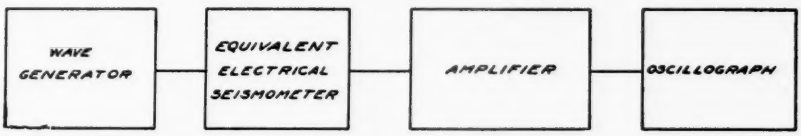

FIG. 1. Schematic connection diagram of laboratory apparatus.

seismometer, but also other reactions, such as, the reaction of the electrical current on the seismometer and the reaction of the seismometer on the electrical circuit.

These facts will be made evident in the mathematical development and experimental results which follow.

SEISMOMETER EQUATIONS

Before discussing the equivalent electrical circuits the differential equation of a seismometer will be developed. For the sake of clearness and simplicity, the equation will be developed for one type of seismometer only. The type selected is shown in Fig. 2. The equations for any other reluctance or dynamic type are similar and can be readily obtained if desired. The force exerted on the sprung mass due to its acceleration is:

$$F = -m'\left(\frac{d^2x_s}{dt^2} + \frac{d^2x}{dt^2}\right) \tag{1}$$

where

F = force on sprung mass,

m' = sprung mass,

[2] F. A. Firestone, The Journal of the Acoustical Society of America, Vol. 4, 1933, p. 249.

x_s = displacement of seismometer case with respect to fixed space,
x = displacement of sprung mass with respect to seismometer case,
t = time,

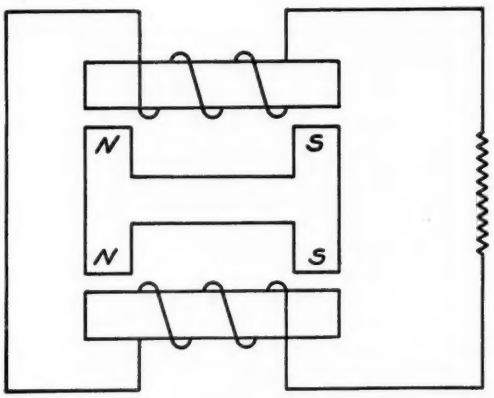

FIG. 2. Reluctance type of transducer.

This acceleration force on the sprung mass is equal to the sum of the forces exerted by the spring, by the mechanical damping friction, and by the magnetic flux, i.e.,

$$F = F_s + F_f + F_m, \tag{2}$$

where F_s = force due to spring (pounds),
F_f = force due to mechanical friction (pounds),
F_m = force due to magnetic flux (pounds),

Now $$F_s = S'x, \tag{3}$$

$$F_f = r' \frac{dx}{dt} \tag{4}$$

$$F_m = -\left[\frac{4\phi_g^2 x/l_g'}{3.6 \cdot 10^7 A_g} + \frac{4\phi_g L' i \cdot 10^8/n}{3.6 \cdot 10^7 A_g}\right] \tag{5}$$

where S' = spring stiffness (pounds/inch),
r' = coefficient of friction (pounds/inch/sec.),
ϕ_g = average flux per gap (maxwells),
x = displacement of sprung mass with respect to case (inches),
l_g' = effective length of air gap (inches),

$L' = L - L''$,

L = total inductance of seismometer (henrys),

L'' = component of inductance of seismometer due to leakage flux (henrys),

A_g = Area of air gap (sq. in.)

It will be noted that in the expression for F_m (Eq. 5) the first term represents the force between the case and the sprung mass which would result from the distribution of the flux in the air gaps if no current were flowing in the seismometer coils and the second term represents the force resulting from the change in the flux distribution due to the current. Equation (5) may therefore be re-written in the form:

$$F_m = -(S''x + \phi_g L'i/.09nA_g) \tag{6}$$

where

$$S'' = \phi_g{}^2/.09 \cdot 10^8 A_g l_g'$$

and is the stiffness of the spring action exerted by the air gap flux when no current is flowing in the seismometer coils.

Now equations (1) to (6) may be combined into a single equation:

$$S'x + r'\frac{dx}{dt} - (S''x + \phi_g L'i/.09nA_g) = -m'\left(\frac{d^2x_s}{dt^2} + \frac{d^2x}{dt^2}\right). \tag{7}$$

It will be noted that equation (7) contains three quantities which are variable with time; namely, x, x_s and i. It therefore remains to eliminate one of these variables. The quantity x will be eliminated by obtaining an expression for x in terms of i. It will be assumed that the seismometer is operating into a resistance load.

Then

$$e = iR = -n\frac{d\phi_c}{dt} \cdot 10^{-8} \tag{8}$$

where

e = voltage induced in seismometer coils

R = total resistance of seismometer and external circuit

n = total turns in seismometer coils

ϕ_c = flux linking one coil

$$= \phi_g(1 + x/l_g') + Li \cdot 10^8/n. \tag{9}$$

The first term in equation (9) represents the component of flux threading the coil due to the magnet and the second term represents the component of flux due to the current in the coil. Now equations (8) and (9) may be combined to obtain the desired relation between x and i.

$$x = -(l_g' \cdot 10^8/n\phi_g)\cdot\left(Li + R\int_0^t idt\right). \qquad (10)$$

The desired differential equation of the seismometer may be obtained by substituting equation (10) in equation (7) and re-arranging:

$$\frac{d^3(iR)}{dt^3} + (R/L + r'/m')\frac{d^2(iR)}{dt^2} + \left[\frac{S'L - S''(L - L') + r'R}{m'L}\right]\frac{d(iR)}{dt}$$

$$+ [R(S' - S'')/m'L](iR) = -(R\phi_g n/Ll_g' \cdot 10^8)\frac{d^3x_s}{dt^3}. \qquad (11)$$

This equation will be used later in connection with the designing of the seismometer equivalent electrical circuit.

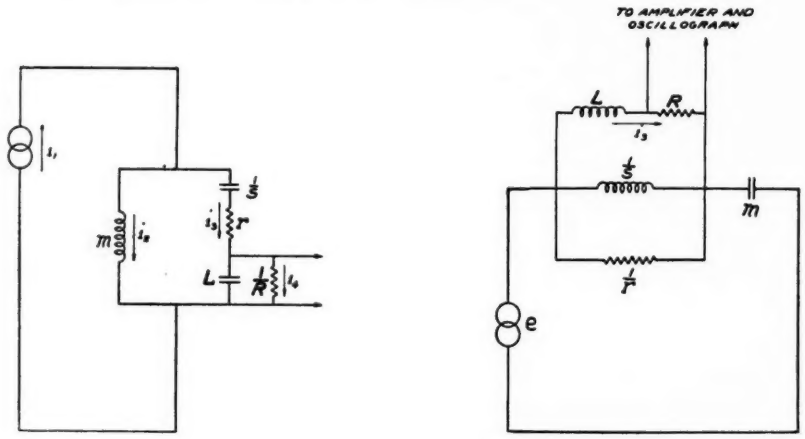

FIG. 3a. Conventional equivalent electrical circuit. i_1 = Velocity of case; i_2 = Velocity of mass; i_3 = Velocity of mass with respect to case; i_4 = Voltage out.

FIG. 3b. New equivalent electrical circuit. e_1 = Velocity of case; e_m = Velocity of mass; $e_{1/S}$ = Velocity of mass with respect to case; e_R = Voltage out.

SEISMOMETER EQUIVALENT ELECTRICAL CIRCUITS

The conventional equivalent electrical circuit of a reluctance or dynamic type of seismometer is shown in Fig. 3a. This circuit is not adapted to the experiment under consideration since the electrical inductance of the seismometer is represented by a capacity, the external load resistance or impedance is represented by the inverse of that resistance or impendance, and the output voltage is represented

by a current. This inverse feature makes it impossible to connect the actual amplifier and oscillograph to the conventional equivalent electrical circuit. The type of equivalent circuit shown in Fig. 3b is the inverse of the conventional circuit. It will be noted that the electrical inductance of the seismometer is represented by that inductance, the external load resistance or impedance is represented by that resistance or impedance, and the output voltage of the seismometer is represented by that voltage. These facts enable one to connect the actual amplifier to the output of the equivalent electrical circuit just as though it were the actual seismometer. Consequently, all that is necessary to obtain a record is to transform the assumed or recorded ground velocity into a voltage "e" by means of the wave generator and apply this voltage to the circuit as indicated in Fig. 3b.

In order to design the equivalent electrical circuit so that it will duplicate the action of the seismometer, use is made of the differential equations of the equivalent circuit and of the seismomoter. The equation of the circuit shown in Fig. 3b can be easily obtained from Kirchoffs' laws and is:

$$\frac{d^3(iR)}{dt^3} + (R/L + r/m)\frac{d^2(iR)}{dt^2} + [(SL + 1 + rR)/mL]\frac{d(iR)}{dt} + (RS/mL)(iR) = (R/L)\frac{d^2e}{dt^2}. \tag{12}$$

The action of the seismometer and its equivalent circuit will be identical if the coefficients of Eq. (12) are made equal to the coefficients of Eq. (11). The relations which result from equating these coefficients are:

$$R = R \qquad r = Ar' \qquad A = 1/L'S''$$
$$L = L \qquad S = A(S' - S'')$$
$$m = Am'$$

$$\frac{d^2e}{dt^2} = -(n\phi_0/l_0' \cdot 10^8)\frac{d^3x_s}{dt^3}$$

where

R = total resistance of seismometer and external circuit,
L = total inductance of seismometer,
L'' = component of inductance of seismometer due to leakage flux

$L' = L - L''$
r' = coefficient of friction
S' = spring stiffness
S'' = stiffness of spring action exerted by the magnet flux
m' = sprung mass

These relations are, therefore, the design formulas for the equivalent electrical circuit.

EXPERIMENTAL RESULTS

Experiments were first made to determine the accuracy of this method in duplicating field records. Fig. 4a shows a record taken in the field and Fig. 4b shows a corresponding record taken in the laboratory. In taking the field record all seismometers were planted together in a trench. Traces 2 to 5 were taken with instruments which recorded accurately the ground velocity. Traces 6 and 7 were taken with instruments which have considerable frequency discrimination. Trace 3 was then transferred by a photographic process to a disc. This disc was inserted in the wave generator and the resulting generator output voltage is shown on Trace 2 of the record in Fig. 4b. It will be noted that this trace is identical to Trace 3 of the field record. Trace 5 of the laboratory record was obtained by applying the wave generator voltage to an equivalent electrical circuit corresponding to the seismometers used for Traces 6 and 7 of the field record. The output voltage of the equivalent circuit was applied to the same amplifier and oscillograph as was used in obtaining Traces 6 and 7 of the field record. It will be noted that Trace 5 of the laboratory record is practically identical to Traces 6 and 7 of the field record. This proves that the equivalent circuit accurately represented the action of the field seismometer.

It is clear, then, that if a voltage representing the velocity of any ground motion is applied to this equivalent seismometer, which is in turn connected to an amplifier and oscillograph, the record obtained will be identical to one obtained by applying the actual motion to the actual seismometer connected to the same amplifier and oscillograph. Consequently, we have an apparatus which in effect allows us to make a field shot right in the laboratory.

There is one other point of interest in connection with Fig. 4. In the design of the equivalent circuit no account was taken of the reaction of the seismometer on the ground. If this reaction had been of any consequence the field record would not have been reproduced

so faithfully in the laboratory record. This experimental procedure, therefore, provides a means of determining the effect of the reaction of the seismometer on the ground.

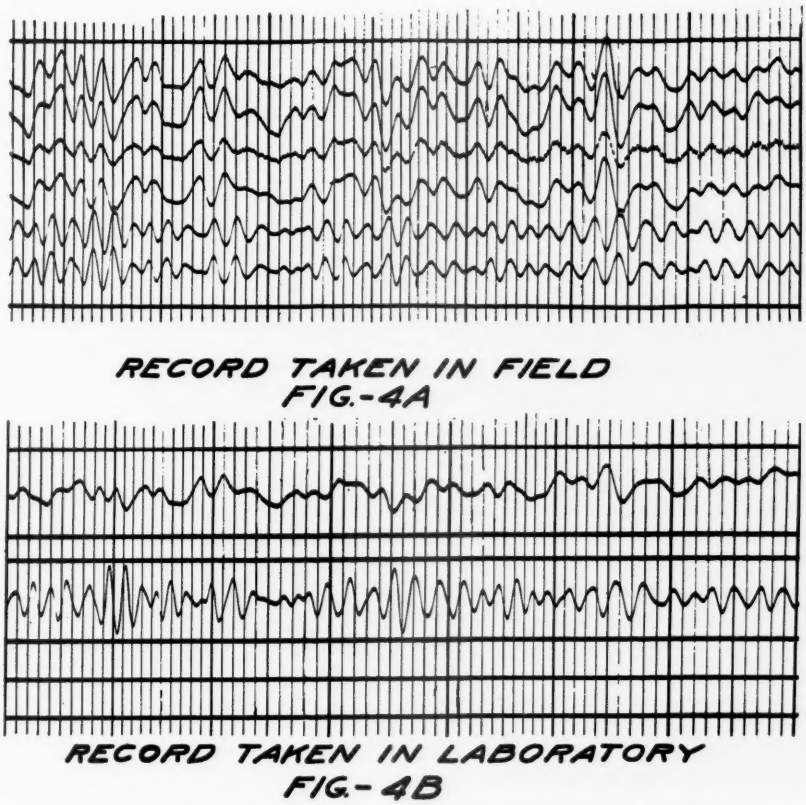

FIG. 4a–4b. Records showing accuracy of laboratory method in reproducing field tests on seismic apparatus.

DISCUSSION

The ability to take field shots in the laboratory is a very valuable tool in determining the merit of any proposed set of seismic instruments or any proposed change in a set of instruments. To make a comparison of one proposed set with another, it is only necessary to take records with each set using wave generator discs corresponding to the types of localities under consideration. Now, the ground motion from a given wave generator disc is always the same, whereas, in the

field the ground motion will vary from shot-to-shot even when taken in the same hole and with the same size of charge. This makes comparison of laboratory records easier and in many respects more certain than comparison of field records.

The laboratory method has another decided advantage when it comes to determining the effect of any changes in the seismometer characteristics. For example, to change the seismometer damping it is only necessary to turn a rheostat in the equivalent electrical circuit and to change the frequency merely requires a change in the inductance and condenser sizes. Even if it is desired to determine the characteristics of an entirely different type of seismometer, its equivalent electrical circuit is easily and quickly built, whereas, it might take considerable time to develop and build a satisfactory seismometer of the new type. The proposed laboratory method in effect allows one to determine the merits of a new type without necessitating its actual construction.

Without any further elaboration, it is apparent that this apparatus is a powerful tool which can be used to determine the merit of any proposed seismic instruments.

In conclusion, I wish to express my appreciation to the entire laboratory staff, and especially to Messrs. R. T. Cloud, David Sheffet and Joseph Eisler for their valuable suggestions and aid in carrying out the experimental work.

EARTHQUAKE INVESTIGATIONS*

MARION H. GILMORE†

ABSTRACT

Earthquakes are a menace to life and property, but they are also a source of important scientific knowledge, for they give us information about the interior of the earth which we can obtain in no other way. The United States Coast and Geodetic Survey has developed instruments for the recording of strong earth motions; has recorded tilt of large areas; has determined dominant periods of buildings, of water towers, of dams and of the ground; and has made special studies of a statistical nature about these phenomena.

History informs us that the Chinese felt earthquakes at least four thousand years ago, but the historical record of these shocks is unreliable and incomplete. Since the beginning of the Christian Era, approximately eight hundred earthquakes strong enough to destroy cities and towns have occurred on land. History shows that an average of thirty thousand persons have been killed each year during the past two centuries by these phenomena. More than twenty thousand earthquakes occur each year; a widely recorded shock each fourteen hours; and a destructive earthquake each six and one-half days.

The appalling loss of life and property during the San Francisco earthquake of 1906, the Santa Barbara earthquake of 1925, and the Long Beach earthquake of 1933, together with recent earthquakes in other countries, has centered the attention of interested persons on the problems presented by earthquakes. These problems have been attacked by governments in all of the seismic regions of the earth, especially in Japan and in the United States. In this country earthquake investigations were carried on by the U. S. Weather Bureau up to 1925. In that year Congress transferred all official seismological investigations to the Coast and Geodetic Survey. It was soon realized that the seismological program should include observations which would aid in the saving of life and property during major earthquakes. To date, these investigations have taken several important directions —namely, recording of strong earthquakes, the study of the vibrations of buildings and of the ground; the recording of the tilt of large areas; the gathering of statistical data about earthquakes and other more special studies.

* Read at the Annual Meeting at Los Angeles, March, 1937.

† United States Coast and Geodetic Survey, Los Angeles, California.

In a close examination of the damages caused by recent earthquakes it was immediately apparent that nearly all lives lost were due to the collapse of poorly constructed buildings or structures not designed to withstand earthquake forces and to fires greatly accelerated by the failure of gas and water supplies. To prevent this loss of life and property damage during a severe earthquake, it was realized that one outstanding need was for better and safer building construction. However, before the engineer would be able to build a structure that would be reasonably earthquake-resistant, he would have to know, as nearly as possible, the forces which a structure must withstand during a severe earthquake. The obtaining of these data and making them available to all interested persons is, therefore, the main purpose of the operation of the strong motion seismographs. In order to secure this information, it is necessary to obtain records of acceleration, displacement and duration, and the periods of vibration during destructive earthquakes. The actual motions of the earth that are measured will be understood best with a knowledge of the mechanics of an earthquake.

Seismologists consider the earth as an elastic solid in which a disturbance in the outer crust sets up vibrations that can be measured by seismographs. Slipping of great rock masses along a fault plane is sufficient to set up these vibrations. It is known that stresses are constantly being built up in the crustal structure of the earth and that at times the material, being sufficiently plastic, will revert to a condition of no strain and, in this manner, prevent earthquakes. On the other hand, when the portion of the crust that is under strain is rigid and resists these bending forces, or when the stresses accumulate faster than they can be absorbed by the plasticity of the earth's crust, the ultimate breaking limit of the rock is exceeded and then at some place the structure will snap.

When there is movement of great masses of rock along a fault plane seismic waves are propagated to all parts of the earth. Several types of these waves are recorded on seismograms, each type having its own peculiar travel time and kind of motion. The seismologist is able to calculate the distance and direction of epicenter from the seismogram by taking into consideration the magnitude and direction of the first motion of the ground and the speeds especially of the first and second preliminary waves which appear on the record.

Some type of pendulum is an essential part of most seismographs. When the supports of the pendulum are forced to move with the

earth, the bob moves in a manner which is rather complex when compared with the motion of the earth. This differential motion of the pendulum and earth is usually magnified and recorded on the seismogram, and from it one is able to estimate the actual movement of the earth particle that is holding the pendulum supports.

Fig. 1 shows a record obtained of a distant earthquake. The P (preliminary) wave is the first to be recorded on the seismogram. Its period is usually short and the amplitude small. Because the displacement of the earth particle is in the direction of propagation, it is similar to a sound wave. The S (secondary) wave is next to reach the seismograph and its period is a little longer and the amplitude is somewhat larger than those of the P wave. The P and S waves follow

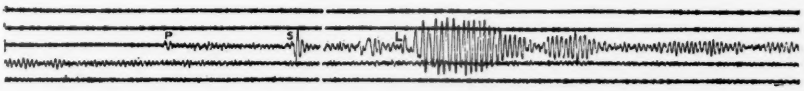

FIG. 1. Record obtained on seismograph 5500 kilometers from epicenter, showing clearly defined normal P, S and L phases. $T_0 = 12$s., $V = 150$.

the path of least time through the body of the earth. The L (surface) waves arrive at the seismograph station at a later time (or interval), partially because they are confined to the earth's surface where the speeds of propagation are lower and the travel times are longer. The L waves may be divided into two groups—the Love and Rayleigh waves. In the Love wave the motion of the earth particle is horizontal and transverse to the direction of propagation. In the Rayleigh wave it moves in a vertical ellipse oriented in the direction of progress. The Love wave is usually faster and contains the longer periods; the Rayleigh waves usually have the greater amplitudes.

A number of very sensitive instruments have been developed to record these teleseismic waves, and some of these seismographs have been made in the United States. These teleseismic records give us valuable information, probably obtainable in no other way, about the interior of the earth and the mechanics of wave motion in the earth's crust.

The teleseismic records give us very little information that is of practical value to the engineer and architect in designing earthquake-resistant structures. They have to know what actually happens at the center of the earthquake. In order to obtain this practical information, there have been developed three types of seismographs by the Coast and Geodetic Survey—namely, the accelerograph, the dis-

placement meter and the Weed strong-motion seismograph. The first two named are for photographic recording, while the latter instrument makes a record on a smoked plate. The accelerograph and displacement meter have in common a drum that holds the photographic paper, a pendulum starting device, a time-marking clock and two six-volt storage batteries. The accelerograph, Fig. 2, has three accelerometers set at right angles to each other to measure the vertical, longi-

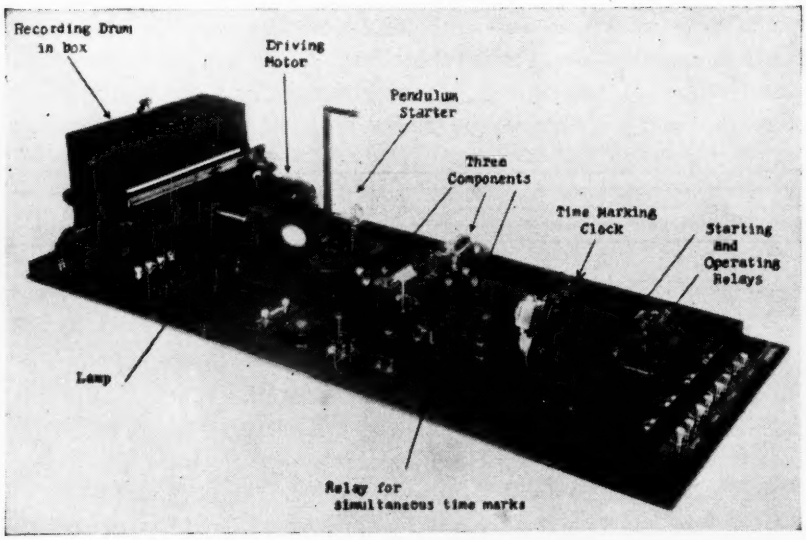

FIG. 2. An accelerograph, with cover off.

tudinal and transverse movements of the seismograph pier. The natural period of the accelerometers is one-tenth second. An accelerometer is a seismograph whose natural period is one-fourth (or less) of the period of the earthquake wave to be measured. At the time of designing the accelerograph it was thought that destructive waves would have a period of at least four-tenths of a second, but recent earthquakes have shown that destructive waves may be much shorter than this period. Since it would be too expensive for these instruments to be in continuous operation, it has been necessary to equip them with automatic starting and stopping devices. An earthquake of suitable intensity will actuate the starter, which in turn starts the drum rotating and the clock flashing time marks on the photographic paper. This continues for a predetermined length of time, usually about seventy-five seconds, at which time the instrument stops (unless the

earthquake is still in progress). It is then ready to record the next shock. One roll of photographic paper is long enough for seven complete records.

The displacement meter is shown in Fig. 3. In order to measure displacement, an instrument should have a free period of at least four

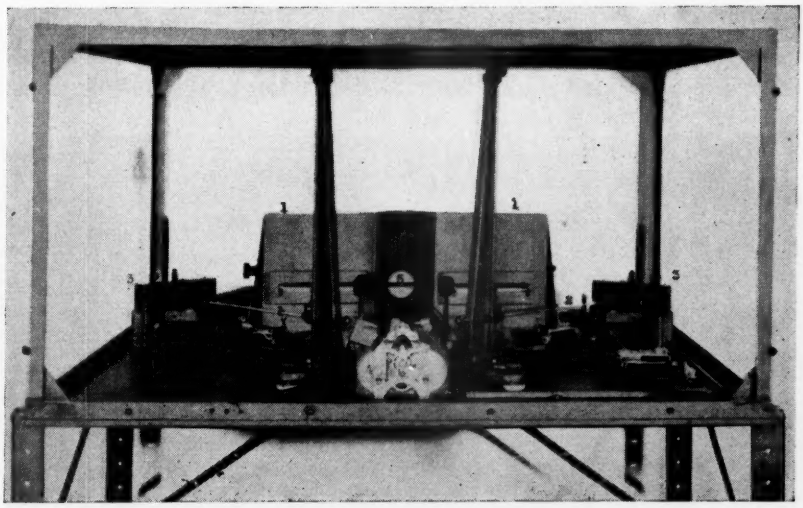

FIG. 3. Displacement meter, with cover off showing (1) recording drums, (2) pendulums set at right angles to each other, (3) oil damping wells and (4) time-marking clock.

times the longest period of the earthquake wave, and for that reason this instrument has a period of ten seconds. It has two horizontal pendulums set at right angles to each other. In most cases it is possible to determine ground displacement from the acceleration curves by means of double integration by a mechanical method developed by Mr. Frank Neumann of the Washington office of the Coast and Geodetic Survey.

The Weed instrument is shown in Fig. 4. It is less complicated than either of the other two mentioned, less expensive to operate, and also slightly less accurate. However it can be used in places where the displacement meter and the accelerograph cannot, because it is smaller and because a light-proof room is not needed for it. The Weed instrument has a free period of about two-tenths second, and measures only the ground movement in the two horizontal directions at right angles to each other. It has a clock that pulls a smoked plate along at very

nearly a constant speed. At the same time the record is being written on the bottom of the plate. This instrument is also equipped with an automatic starting device, but the rest of the instrument is not as positive in its action as the other two instruments.

From the records of strong earth motions obtained with these instruments it is possible to determine the acceleration, displacement, period and duration of the actual ground movements. When these elements are known one can calculate the actual forces involved. It is then possible for the engineer to determine how strong he must

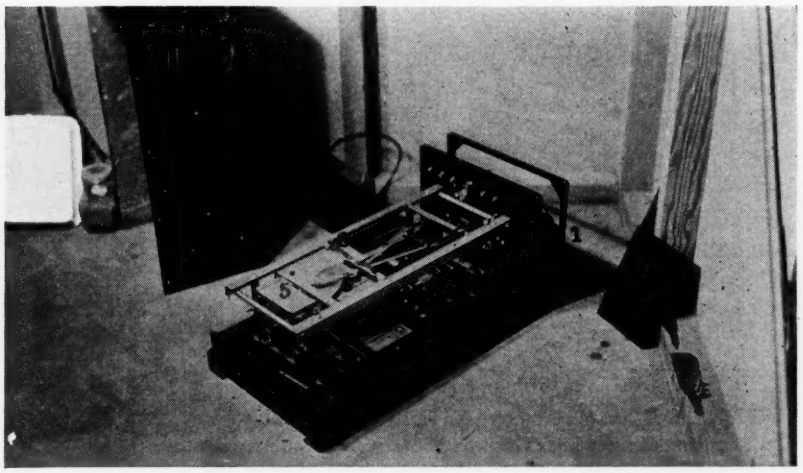

FIG. 4. Weed seismograph, showing (1) batteries, (2) plate holder, (3) recording arms, (4) Braunlich starters, (5) driving clock and (6) inverted pendulum supported by three wires.

make a structure so that it will be able to withstand the forces of such an earthquake.

This information is essential to proper designing; but by itself, it is not enough. It is necessary to know something about the fundamental vibration period of buildings. It has been known for some time that each structure vibrates in its own free period and this information should be obtained for as many structures as possible in every earthquake region. If the period of the building is the same as the ground period set up by most earthquakes in any particular region a very dangerous resonance would be built up. Resonance between the ground and structure for a very short time might build up large amplitudes of motion, increasing the destructiveness of an earthquake

shock. It is, therefore, desirable to secure more data on the periods of buildings and of major earthquakes in the region in order that future structures might be built that will have fundamental periods different from the ground periods set up by the earthquakes. If this should not be practical, then it would be necessary to make the structures stronger so they will be able to withstand the added strain of resonance.

Buildings are caused to vibrate in their own free period by the wind, by nearby traffic, by machinery, and by earthquakes. Sometimes, however, it is necessary to set the building vibrating in order

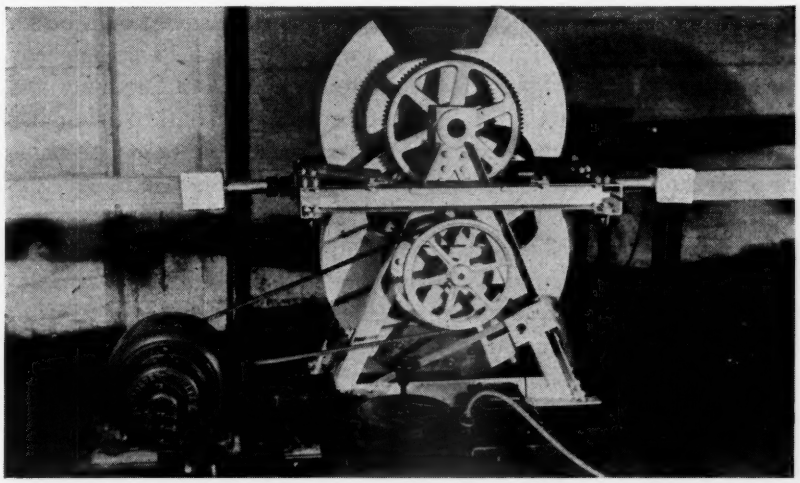

FIG. 5. The building and ground vibrator, set for exerting horizontal forces.

to secure its free periods. For this purpose a machine has been built for putting controlled vibrations into a building. This machine, shown in Fig. 5, has three one-fourth inch steel discs each thirty inches in diameter, to which are bolted lead plates to produce the desired unbalance. Two of the discs rotate in one direction and the other in the opposite direction. It is capable of putting about three thousand five hundred pounds of force into a building or into the ground. When the period of the rotating discs is the same as that of the structure being tested, vibration meters which are set up in the building will record a maximum amplitude.

The instrumental period of the vibration meter in use at this time is usually set well above the expected period to be measured. The

fundamental period has been measured in several hundred buildings and for many water towers and dams. In order to check buildings for damage, their periods should be redetermined after a severe earthquake. If the period of the structure has been materially lengthened by the shock, the building probably has suffered considerable structural damage.

After the average periods for destructive earthquakes have been determined in a region and the periods of a number of buildings are known, it is then necessary to determine the dominant period of the ground, if at all possible. It is reasonable to conclude that if the period of the ground, the period of the building, and the period of the earthquake are nearly the same, a more dangerous resonance would be set up in a structure than if these periods were different. There has been considerable work and study on this problem in both Germany and Japan. Recently, the United States Coast and Geodetic Survey has undertaken this vital problem. A party has been working in California for several months, securing data on a group of Federal Buildings, and recently working upon the problem of ground vibrations. A very sensitive vibration meter has been used and controlled vibrations have been put into the ground with the shaking machine, Fig. 5. Explosives have also been used with much success. This experimental work has accomplished a great deal. Two important facts have been demonstrated in the work—namely, that a shaker capable of putting more energy into the ground may be necessary and improvement in the sensitive vibration meter is called for before much additional work is undertaken.

Ground periods have been determined in the Leine Valley in Germany and in several places in Japan. In California, Dr. Gutenberg has measured the dominant periods in many earthquake records and found the following periods to be in excess of all others: two- to three-tenths second, five- to six-tenths seconds and one second.

For many years earthquakes have been associated with certain crustal movements, such as block-tilting, arching and slipping along fault planes. It has been noticed in Japan, in Germany and in the United States that certain areas in regions subject to earthquakes have had considerable movement in recent times, not only during an earthquake but also during the interval of time between shocks. For this reason a knowledge of geological conditions is needed, both of the surface and of the subsurface. The location of all active faults, especially, should be known. Some of this information can be ob-

tained from a study of surface conditions, but a great part must come from mines, oil wells and the several geophysical methods of studying geological features. The seismic method, suggested by a study of wave propagation in earthquakes and used rather extensively during the World War, is capable of outlining underground structures. Refraction and reflection shooting are two of the methods used in locating oil-bearing structure. These methods have been developed principally for economic reasons in the search for geological formations that are favorable for the accumulation of oil and minerals. However, geophysical methods can be used strictly for geological investigations, and are being used in that manner in several states. The records obtained of strong earthquake motions are very similar to those secured in geophysical prospecting, and the interpretation is about the same. In the seismic method of prospecting there is an advantage over the records obtained from an earthquake in that the exact time of the explosion and the place are known. In the earthquake wave, neither the time nor the place is known; both must be secured from the records. When the time and place of the explosion are known, the exact travel-time in the different formations can be secured and this information may then be carried over to the earthquake field. There are other geophysical methods that are very promising to the seismologist—namely, methods that make use of the magnetic or electrical properties of rocks, and of the variation in the force of gravity. By the use of the magnetometer, the San Andreas fault has been traced for a considerable distance into the Imperial Valley where it is covered to a great depth by sediments. It is likely that the other geophysical methods will gradually be applied to the problems in the study of earthquakes.

The Coast and Geodetic Survey has been studying crustal movements in two important ways. On the campus of the University of California at Berkeley four tiltmeters, using the principle of interferometry, are located on either side of an active fault. The measurement of earth-tilt was started in Japan, and may lead to some important findings concerning the movements continuously in operation on opposite sides of an active fault. These tilt-meters are located in pits about twelve feet deep. A shallow dish containing oil is covered with a (circular) glass plate, ground in such a way that the two plane sides are not quite parallel. The surface of the oil is level and the dish is adjusted so that the lower surface of the glass cover is nearly level. The reflected beams of a ray of monochromatic light

normally incident on the two surfaces will re-inforce or interfere with each other, depending upon the location of the points of incidence, so as to produce alternate light and dark bands of light, or interference fringes. The number of these fringes is proportional to the angle between the oil surface, which is level, and the lower surface of the glass plate which is subject to the change of tilt of the earth as it is an integral part of the earth. The instruments have been calibrated so that a change of one fringe within the definite confines of the field corresponds to one second of tilt. The direction of tilt can be determined from the direction of curvatures of the fringes. Three tiltmeters are equipped for visual reading, on which daily readings are secured; on the other instrument an automatically operated camera photographs the fringes once each hour.

The second method of determining block-tilting is by very precise leveling and triangulation. It has been found in Japan that considerable crustal movements are continuously in progress, mostly vertical changes of large areas. There, the best method of obtaining the amount of change is by leveling, which is much less expensive than triangulation work. The first work done in this country to determine surface movements in an area was after the earthquake of 1906 in San Francisco. Even though this first work did not have the desired accuracy, it was conclusively determined that relative ground displacements had occurred and that the amount of movement rapidly decreased as one moved away from the San Andreas rift.

The method used in California to determine the amount of tilt is by geodetic measurements. A large number of stations are established in the vicinity of an active fault, each point being marked by a concrete monument, and the location and elevation being detertined by the most precise methods known. The locations and elevations are re-determined periodically, thus showing the amount of change that has taken place. Some vertical change has been noted in the Imperial Valley and it is known that the earthquake in Utah in 1934 produced some changes in the elevations of bench marks over an area of several hundred square miles. Records of precise leveling have been kept for some time in Japan and seismologists there have made an important discovery. The records date back far enough so that they have data of crustal movements through the last three major earthquakes. These data have been plotted and a decided similarity exists between the curves during the first and second and the second and third major shocks, thus pointing to the fact that changes in elevation are very important. Earth stresses are con-

tinually being accumulated in the crust, gradually changing the elevation of the surface, until such a time as a break occurs. If this crustal movement can be accurately measured between several major earthquakes over a long period of years, it is reasonable to expect that important predictions can be made. However, the time has not yet arrived when earthquakes can be predicted.

The non-instrumental program of the United States Coast and Geodetic Survey is one of great importance in the study of the effects of earthquakes upon man, animals, and property. This type of work is one in which many cooperative observers, in all parts of a region in which an earthquake is felt, send the Coast and Geodetic Survey their reports on post-cards furnished them for that purpose. One of the first problems in this work was to devise some kind of intensity scale that would be suitable. Many scales have been devised in the past, but not one has been entirely satisfactory. The two best intensity scales now in use are known as the Modified Mercalli Scale and the Rossi-Forel Scale; the latter divides the total intensity of earthquakes into ten equal parts, the former into twelve parts. These intensities start with an earthquake that is recorded on only one or two seismographs or by only the very experienced observer, and end with earthquakes causing complete and total destruction. The questions on the questionnaire cards are so arranged that the answers enable an experienced seismologist to determine the intensity of the shock at all points in the region and to draw the iso-seismal lines accurately.

Non-instrumental determination does not always agree with the instrumental location of earthquakes because of geological conditions. The iso-seismal lines drawn for an earthquake occurring along a known fault nearly always take an elliptical form with the long axis parallel to or coinciding with the fault. Earthquakes due to volcanic origin usually take a circular form. In most cases where the iso-seismal lines are very complex, the reason can be attributed to some sub-surface geological characteristics.

A map of an earthquake has many important uses. It enables one to obtain the relation of intensities at different points and to secure an idea of the extent of the shock. One is able to see the relative amounts of damage to structures that are on different soils, whether the foundation was on solid rock or on made land. Probably the most important use of such statistical information is for the purpose of insurance. The only manner in which the insurance risk can be appraised and the rate established in fairness to both the company and owner is

through a knowledge of the facts of earthquake occurrence in the past and the damage caused to particular kinds of structures. The history of earthquakes in the United States is too short and the records contain too little information as yet to form any definite conclusions about the occurrence of earthquakes. The value of this program as a whole increases with the number of reports and with the length of time over which they are received.

Another important non-instrumental study made after each major shock on land is the determination of damage to property. Pictures showing damage to buildings, roads, water tanks and dams are made for future study. In looking over the actual damage after an earthquake it is often possible to see why certain structures have collapsed, and in many cases to observe what could be done to prevent failure in the future. The San Francisco earthquake of 1906 furnishes an outstanding example. There, a lack of water prevented a control of the fires that completed the destruction of the city. The water and gas mains were broken, something that is likely to happen in any city in an earthquake region. San Francisco now has many underground reservoirs of water throughout the city and will be enabled to fight fires should the necessity arise again. It has also been observed that the reason many structures fail during an earthquake is that their different parts are not properly tied together. These are only a few of the many things that are learned from a study of earthquake damage.

In the past the United States has made very few contributions to the study of earthquakes, but at present the problems are being attacked on many fronts and important results are to be expected. Business interests, organized groups of engineers and architects, educational institutions and many noted and distinguished scientists are cooperating with the Federal Government in its attack on these problems. Through this coöperation it is felt that important steps, toward the solution of the earthquake problem, will be made.

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ADVERSE EFFECTS ASSOCIATED WITH VARIABLY COMPOUNDED SEISMOGRAPH RECORDS*

M. MOTT-SMITH†

ABSTRACT

Although the method of variably compounding seismic reflection records, in order to secure so-called "Controlled Directional Sensitivity" has distinct advantages over the more generally used fixed compounding or so-called "Multiple Recording" and other methods, a simple graphic analysis shows that this "variable compounding" also involves the introduction of adverse effects. Among these are a less directional sensitivity, the production of artificial "noise" which is added to that picked up from the the ground, and the magnification of minor waves which the analyzer picks out at random.

This graphic analysis shows that, if the two waves arrive simultaneously from different directions, they can be separated if they differ sufficiently in direction and not too much in amplitude.

Exception is taken to the use of the term "diffraction" as applied to the wave scattered from the edge of a fault.

The method is shown to be superior only in certain special cases, which are, perhaps, rather few.

In the method of multiple recording, as is well known, several geophones are grouped on one channel. The object and the effect of this arrangement is to magnify several-fold the reflections coming up from below, while the ground wave and other surface disturbances coming in the same direction are less magnified. It is obvious that a plane wave coming vertically from below will meet all the geophones at the same moment, and the magnification will be a maximum. For all other directions the magnification will be reduced. For a wave coming approximately horizontally from the shot point, a ground wave, the magnification will be a minimum. Indeed, if there are eight geophones spaced one eighth of the ground wave length apart, and the latter consists of several *equal* cycles, the middle cycles will be entirely eliminated, as shown in Fig. 1. There is, however, a crest and a trough at the beginning and end of the summation wave, each having, for this number of geophones, 2.6 times the amplitude of the original wave. There is still some magnification, but since a reflection coming up vertically will be magnified eight times, there is an advantage of three to one in favor of the reflection.

This figure is taken from Mr. McDermott's recent article in *The Petroleum Engineer* for February 1937, except that another half cycle has been added to the wave so as to show a trough instead of a crest

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at the end of the disturbance. This is a little nearer to nature. Still nearer, would be to use a damped wave. In that case, the elimination of the middle cycles would not be complete, as is shown in Fig. 2. The initial bump is still there and of about the same amplitude, but

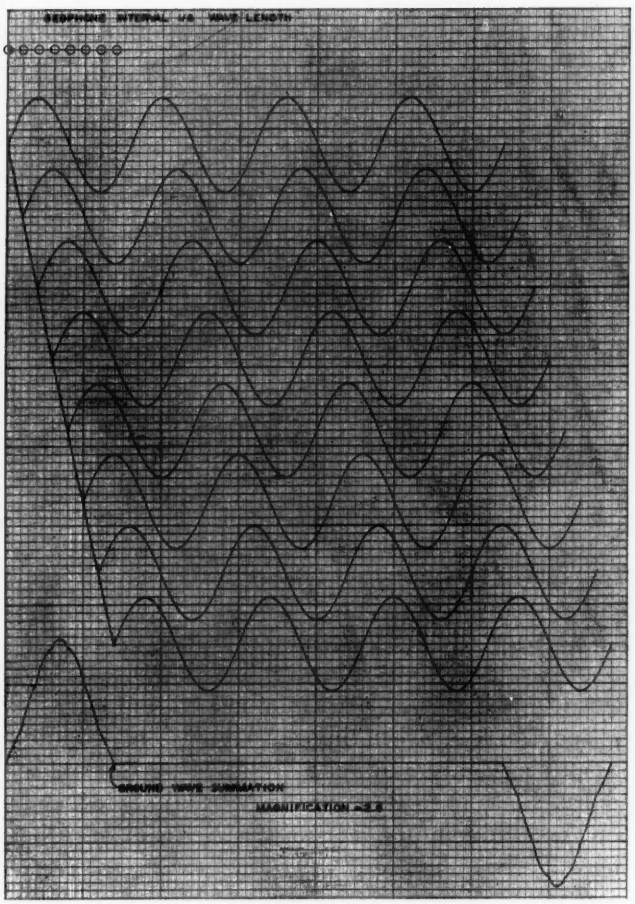


FIG. 1

the final trough has disappeared. Incidentally, one may note the lengthening of the summation waves, which is characteristic of all compounding.

The diagrams apply, of course, only to the one wave length shown. All longer waves will be more magnified, all shorter waves less magnified. The diagrams apply also only to the one direction, that of the

shot line. Disturbances coming in obliquely to the shot line will be more and more magnified, until when they come in at right angles, they will be magnified to the same degree that the vertical reflections are. This is not so good! But at least it is no worse than on the ordinary records where everything is amplified equally.

The method of course does not reduce noise originating below the surface of the ground, except that which arrives at a large angle to the vertical.

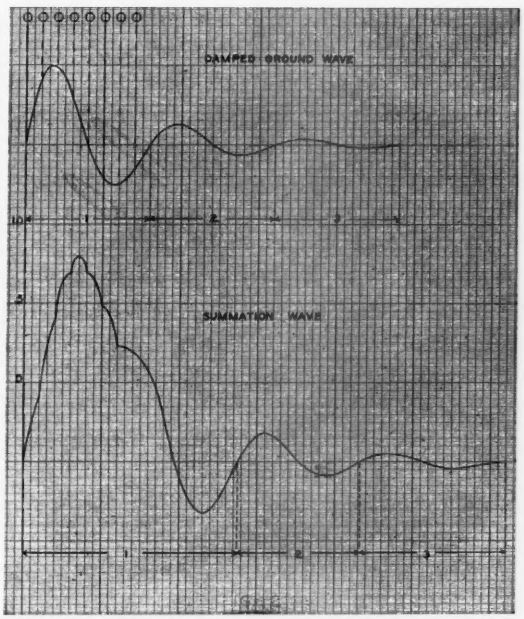


FIG. 2

The multiple hook-up may be called a method of fixed compounding. Its maximum response is for one direction only—the vertical. If its directional sensitivity is low, the amplitude of oblique reflections will not be too greatly reduced, until very steep dips are reached. Hence, in an easy country of gentle dips it may safely be used. But in a difficult country where steep dips, folding, and faulting abound, as in California, it may not be so good.

The low response for steep dips could be remedied if we were able to tilt the row of geophones so as to keep it always perpendicular to the emergent ray. But this cannot be done, and besides requires that

we know beforehand the angle of emergence. Also, on the same record, reflections may come in at different angles.

Nevertheless, a way of accomplishing this variable compounding has been devised by Mr. Frank Rieber, and has been described in various lectures and articles.[1] The compounding is done in the laboratory and on the records, instead of in the field and on the geophones, and it is done on the channels instead of on the groups. The purpose and the effects are somewhat different. The arrangement permits every angle to be tried on every record, so that one does not need to know beforehand the angle of emergence. All are tried.

In the latest arrangement, ten geophones are used, each on a separate channel. The impulses of these geophones are recorded as sound tracks on a moving picture film. The film is then run some twenty times through a machine called the analyzer, where, by means of a slit whose angle can be varied, the ten track records are superimposed, giving thus one composite trace for each position of the slit. All of these twenty traces are drawn by a pen on a single strip of paper. When the slit angle coincides with the step-out angle of a reflection on the sound tracks, that reflection will show a maximum amplitude on the corresponding analyzed trace—or should one say synthetic trace? On all other traces, the wave being out of phase, the amplitude will be diminished. One therefore looks for the trace on which the reflection shows a maximum amplitude and reads the step-out corresponding to that trace.

This is indeed a method of determining the step-out, which is the converse of the usual one. Whereas, in the old way, one finds a reflection and measures its step-out, in this method, one assumes a step-out and searches for a reflection having that step-out.

It is claimed for this reverse process that it can distinguish two waves that arrive at the same time from different directions, which waves on the ordinary record would be so intermingled that neither could be identified. The two waves might come from the two sides of a syncline directly under the instruments, or one might be an ordinary reflection, the other a wave scattered from the elevated edge of a fault. It is claimed that the latter can be distinguished from the former by the fact that its wave front would be more curved. In general, it is claimed that the machine can give definite and inter-

[1] A New Reflection System with Controlled Sensitivity, GEOPHYSICS, vol. I, no. 1, Jan. 1936, p. 97; Visual Presentation of Elastic Wave Patterns Under Various Structural Conditions, GEOPHYSICS, vol. I, no. 2, June, 1936, p. 196.

pretable results in regions so difficult and complicated that the ordinary records are quite undecipherable.

These would indeed be valuable assets. Before examining these special cases, however, let us first inquire whether, for ordinary seismic work, the new method has any worth while advantages over the older ones, especially in view of its greater cost, complications, and consumption of time.

First of all, it may be doubted that the directional sensitivity is as good. The determination of the step-out by locating the position of a maximum is an inherently inaccurate method. Any quantity diminishes slowly at first from its maximum value, because the rate of change at that point is zero. A zero method is preferable, because at that point the rate of change is a maximum. But such a method cannot be applied in this case, because the zero is buried at the bottom of the noise layer. Variable compounding is therefore committed to the location of a maximum, and the only thing that can be done about it is to make the peak as sharp as possible, or perhaps increase the scale of the drawing so that the measurements can be made more accurately.

It is interesting to note that in this regard the interests of multiple hook-up and of variable compounding are diametrically opposed. The former requires *low* directional sensitivity, for otherwise the amplitude of oblique reflections would be too greatly diminished. The latter requires *high* directional sensitivity, in order that a small change in obliquity may be detected. In fact, the advocates of multiple hook-up assert that a dip of twenty degrees makes a quite negligible change in the amplitude, while those who favor variable compounding claim that a dip of one degree makes a very perceptible change. It would seem that both cannot be right.

However, the directional sensitivity is not a fixed quantity. It varies directly with the geophone spread and about as the square of the number of geophones. By adjusting these factors, it may be made high or low, but perhaps not enough to satisfy both parties.

The manner in which the amplitude of a reflection descends as the slit is turned, is shown in Fig. 3, both for five and for ten geophones. The curve for ten geophones starts at twice the altitude of the other curve—the magnification being twice as great—but later merges with the other curve. The additional geophones affect only the first part of the curve. In either case, the amplitude does not descend below that of the original wave. The signal was assumed to be a single cycle

of a sine wave—a rather artificial assumption. If the signal were a continuous wave, the results would be as shown in Fig. 4. The amplitude now descends to zero, but rises again to a maximum in reverse phase. The curve for ten geophones is now not only twice as high as the other, but is half as wide at the base. The gain is four-fold. Actually, the signal will be a damped wave train and the results will lie between those shown.

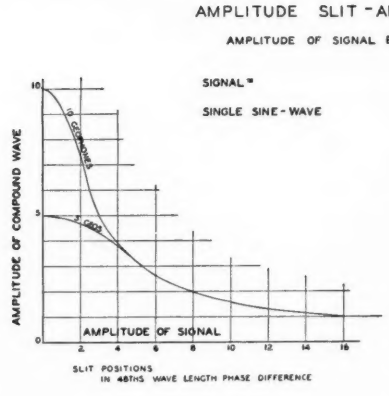


FIG. 3

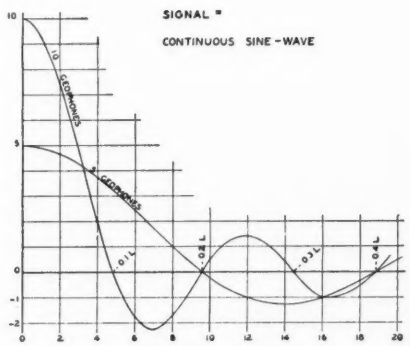


FIG. 4

In an article on the Geo-Sonograph in the February issue of *The Petroleum Engineer*,[2] the analyzed traces of the machine are shown to differ by a step-out of .005 seconds. By estimating between the traces the step-out is determined to .001 second, which is as good as anybody can do. But a step-out, which in one figure of the article is marked as −.032 seconds, is shown in another figure to correspond to a dip of nearly 60 degrees. Of course, the figure may for the sake of clearness be exaggerated. But at any rate, on the ordinary records a dip of that magnitude would show a step-out four or five times as great. This step-out, being likewise measured to a thousandth of a second, would give a directional sensitivity four or five times as great. Furthermore, it is said that to locate a maximum accurately on the analyzed traces, a difference in amplitude of a hundredth of an inch must be detected. To determine a step-out on the ordinary records to a thousandth of a second requires the end points to be located to only a fiftieth of an

[2] Applications of the Geo-Sonograph to Petroleum Exploration, The Petroleum Engineer, VIII, Feb. 1937, p. 140.

inch, and no measuring has to be done. One may therefore seriously doubt that the directional sensitivity of the new method is as good as that of the older ones.

Now this is a serious defect. The step-out is really the weak point in dip shooting. It is a small quantity, consequently cannot be measured with accuracy. All the other quantities that enter can be measured with far greater accuracy. Every effort should therefore be made to strengthen this weak point. Any device that does so is a step forwards, and any device that adds to its weakness is a step backwards.

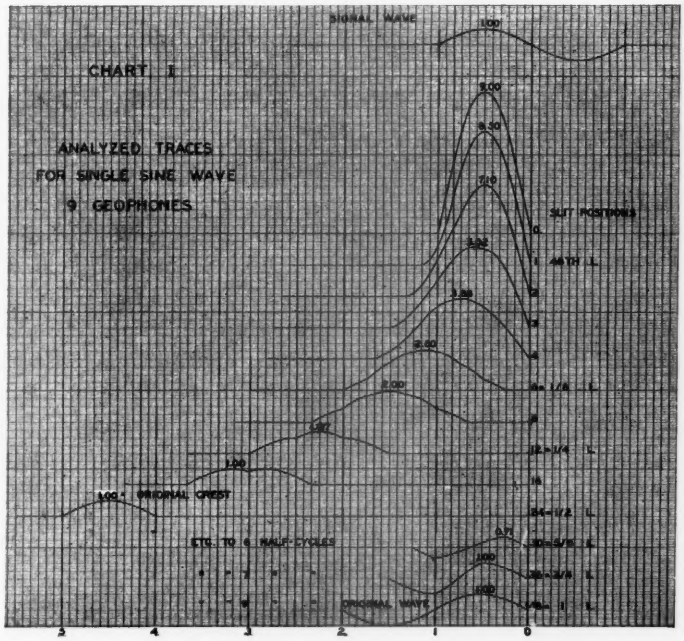


CHART I

What happens to a wave when it is put through the machine? A simple case is analyzed graphically in Chart I. Here, it is assumed that the signal consists of one complete cycle of a simple sine wave—shown at the top. Other than this, there is complete silence on the sound tracks. This is the most favorable assumption that could be made. A short highly damped wave train would be nearer to an actual reflection, and a few stray waves before and after would be nearer to

the truth. But all of these more complicated assumptions would be less favorable than the one chosen.

To simplify the calculations, nine instead of ten geophones have been used. The step-out is assumed to be zero. This, as we shall see later, is the most favorable step-out. Under these circumstances, the signal wave is impressed on each sound track at the same time point, and the vertical slit as it sweeps over them meets them all at the same moment, adds them up in phase, and the resultant wave, nine times magnified, appears on the middle or zero trace of the analyzed record —the first one on the chart. Only the left half of this and the other traces are shown, the right half being the same in reverse phase; and only the traces for positive slit angles are shown, the others being the same in reverse order.

When the slit is inclined, the waves are added up out of phase and the amplitude of the resultant is diminished. But there is another effect to be noticed. The forward end of the inclined slit meets the signal wave first, traverses a part of it, and then a moment later the slit meets the wave on the next sound track, and so on. On leaving the signal, the forward end leaves first, and then in succession all the other sound tracks, until at last the trailing end is clear. For the inclined slit, therefore, a longer time must elapse between the first and last contacts with the signal than for the vertical slit. Consequently the resultant wave is lengthened. The crest moves to the left, the trough to the right, until they become quite separated. The space between is filled with more or less interfering portions of the signal wave. On some of the traces shown on the chart, the central interference is complete. This is because, for convenience of calculation, the slit was turned through equal intervals of phase difference. In practice, these exact points would rarely be struck. Furthermore, the complete interference is due to the fact that the second or negative half of the sine wave is exactly the same as the first or positive half. If, as is the usual case, the signal consisted of a damped wave, the amplitude of the second half would be less than that of the first half, and there would never be complete interference in the middle. The spreading out would be of the same length, but the middle would be filled with a continuous rumble.

On trace 24 of the chart, the phase difference is half a wave length. The hump and hollow are now widely separated with nothing between them, and are reproduced exactly as they were on the original sound track. This is because the forward end of the slit now runs

through a half wave length before encountering the wave on the next track. The negative half of the first wave is then combined with the positive half of the second wave, with complete interference, and so on until only the last half of the last wave remains to be traversed by the trailing end of the slit.

As the slit is further turned, the spreading continues. Waves reappear in the middle, and there is never again complete interference. For a phase difference of a wave length, the original signal is simply repeated on this trace as many times as there are geophones. The leading end of the slit now traverses the whole wave on the first track before encountering it on the second, and so on.

If the signal had consisted of two waves of equal amplitude, this trace would have shown the first wave at the beginning, the second wave at the end, and a series of waves of double amplitude in the middle. The total length would have been one wave length greater than before. In general, the total length of the disturbance for a phase difference of a wave length is $n(C-1)L$, where n is the number of geophones, C the number of cycles in the signal, and L the wave length. This is independet of the wave form.

Going back to the one cycle wave, if the slit is further turned, we get again the same number of replicas of the original wave, but with spaces between them. An interval of time now elapses after the leading end of the slit has traversed the first wave and before the slit encounters the second wave, and so on. Thus the disturbance continues to spread as the slit is turned.

Finally, the length of the disturbance is a minimum for zero step-out. Suppose that for this step-out a phase difference of one wave length were reached on each of the extreme traces, and that for some other step-out a phase difference of half a wave length were reached on one of the extreme traces. Then on the other extreme trace, a phase difference of one and a half wave lengths would be reached, and the disturbance would be correspondingly longer.

Of course, we have used a particular frequency in this analysis. But the frequency does not affect the magnitude or position of the maximum. It affects only the rate at which the waves go through their paces. Thus, if the frequency were doubled, we should, on the trace marked 12 on the chart, arrive at the same state as that shown on the trace marked 24. With increased frequency, the amplitude descends more rapidly from its maximum, so that short waves are an advantage in this method.

We see then that one of the effects of compounding is to stretch the signal wave to many times its original length, and that the amplitude of the disturbance never descends appreciably below that of the original wave. More geophones enable the slit only to pick up the signal more times, to throw more replicas of it into the general noise layer. They prolong, but do not diminish the disturbance produced by this one wave.

What happens to one signal wave happens to all. They are all stretched. And these stretched waves, piled one on the other, raise the general noise level. This artificial noise, which is produced by the machine itself, quite apart from and in addition to the noise that is picked from the ground, is a serious matter. For the noise that is already in the ground, and which masks the reflections we wish to pick out, is one of the greatest obstacles that all seismograph methods have to contend with. Any device that can diminish this noise in relation to the signal strength is a step in advance. Any device that increases this noise is a step in the reverse direction.

But there is another aspect to this noise difficulty. There are occasions when the very analytical powers of the machine may become harmful. The noise in the ground, except for a small part produced by extraneous disturbances, and except for the direct ground wave, and for refractions up from the first few hard layers, is composed entirely of minute reflections. They come in all directions from innumerable underground points. Being reflections, each separate impulse has all the characteristics of a reflection—sudden onset and a regular step-out. But, on the usual record, they are so mixed and blended that these characteristics are not discernible. We see only an irregular wiggly line. We select as a reflection one that protrudes above the general noise level, one that is big and strong enough to be identified on successive traces and thus to show its step-out. In so doing, we are not distinguishing a reflection from other things that are not reflections; we are merely distinguishing an important reflection from others that are not important. This is not difficult so long as the unimportant ones are weak and are so thoroughly mixed as to be quite indistinguishable *as* reflections—which is the case on the usual record.

But the compounding machine is an excellent device for singling out individual voices in the crowd. If the slit happens to come into exact or nearly exact phase with the step-out of a small wave below the noise level, this wave will be magnified above the neighboring

waves, which may be nearly as large, but are out of phase. The slit being turned by intervals and not continuously, it is a mere matter of chance, which of these small waves is selected for magnification.

On the usual record the average height of the noise level is about half that of the best reflections, and many are picked that barely rise above that level. It would be an advantage if the noise level could be reduced, for it is undoubtedly true that many reflections that are still sufficiently important to be taken into account, are lost in the noise layer. But the consequence of this high noise level is, that when one of the small waves that compose it gets into phase and is magnified ten times, it shows a peak quite comparable to that of an important reflection, and may be mistaken for one.

It must be remembered that while compounding increases the signal-to-noise ratio, it does so, not by lowering the noise level, but by raising the signal height. Actually, compounding raises the noise level. Thus, by a formula developed by P. K. Klipsch[3] for completely random noise, ten compounded geophones would raise the average expected noise level to 2.8 times the amount it was for one geophone. But at the same time the signal is magnified ten times. The gain in signal-to-noise ratio is thus 3.6. If the ratio on the original one geophone record was 2, on the compound record it would be about 7.

Compounding therefore does not aid in picking out a wave that is below the noise level. It only makes more conspicuous those waves that already rise above it, and which, presumably, could have been picked, with sufficient scrutiny, without the aid of compounding. The only exceptions are, as mentioned above, the small waves below the noise level that are picked out at random.

For general use, therefore, it appears that the method of variable compounding is not superior to the more usual methods. The directional sensitivity is not as good; there is what may be called machine noise that is added to the random noise from the ground; and finally, it picks out at random obscure waves and makes them look like important reflections.

Let us now examine that feature for which the method is particularly recommended, namely, its ability to separate two waves that arrive simultaneously in different directions. A graphic analysis of this case is made on Chart III.[4]

[3] Some Aspects of Multiple Recording in Seismic Prospecting, GEOPHYSICS, I, 3, Oct. (1936) p. 35.

[4] Chart II has been omitted from this paper.

To the left are the original waves as they would appear on the ordinary record. Each consists as before of one simple sine wave, but one wave has half the amplitude of the other. The major wave has a zero step-out, the minor wave, a large positive step-out. On the first and last of the ordinary traces, the two waves are just clear of each other. On all the others they mix. Even in this simple case, with the added complications of actual practice, they could probably not be distinguished.

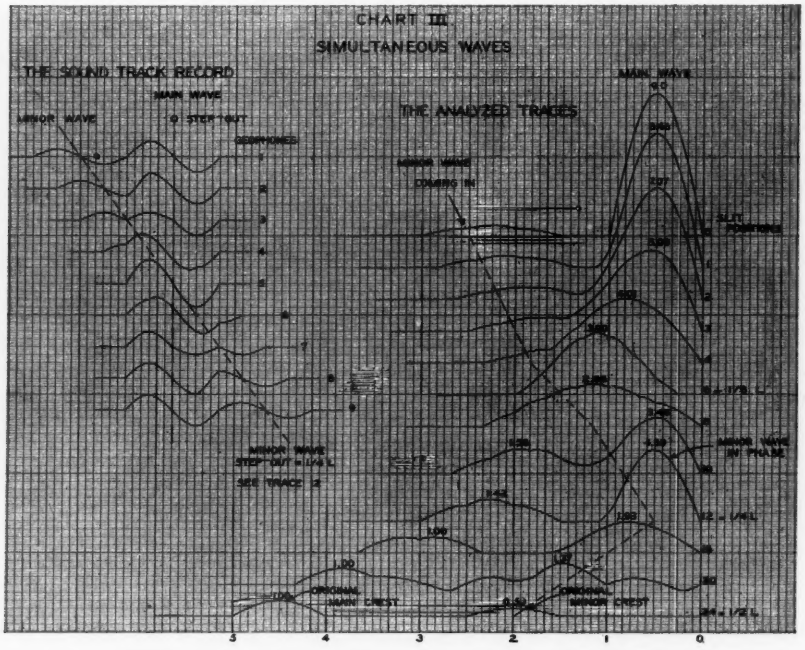


CHART III

To the right are the analyzed traces. On the first or zero trace the large wave shows up as usual. On the following traces, its amplitude diminishes, the crest moves to the left, and then just as it gets clear of the middle, the small wave appears quite suddenly and plainly. On the last trace, corresponding to a half wave length phase difference, the crests of both waves appear, reduced to natural size, and nothing else appears.

Of course, in practice, where each signal consists of a damped wave train and noise is present, things are not so simple as this. But the

two waves could perhaps still be distinguished, if not at this angle, then at a larger one.

On Chart IV the same two waves are shown with a less angular separation. It is now seen that the small wave is combined with the big one and cannot be distinguished from it. The big wave did not move out of the way fast enough. The only thing observable on the analyzed traces is the extreme slowness with which the amplitude of

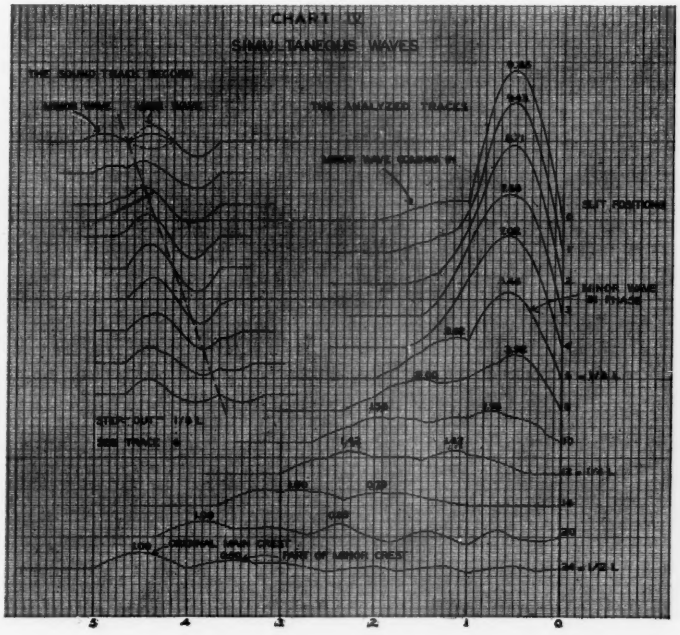


CHART IV

the combined wave diminishes. The drop between the first two traces on the previous chart was 0.054. On this chart it is only 0.02. One must remember that of all that is drawn on these charts, only the first half cycle is visible on the traces as drawn by the machine. The rest is buried under the preceding and following waves, which are here omitted.

The analysis shows, however, that if two simultaneously arriving waves differ enough in direction, and not too much in amplitude, they can in fact be separated, and their individual step-outs determined.

One proposed use of this property is to detect a syncline that is

directly under the instruments. But if one approaches such a syncline by the ordinary methods, he gets a down dip on one side and an up dip on the other, and perhaps confusion in the middle. It is not difficult to supply the missing part. Variable compounding would simply give a few points on that part of the profile which would otherwise have to be extrapolated. If it be objected that extrapolations are always uncertain and inaccurate, one may reply that the seismic explorer is not particularly interested in synclines. He is more interested in anticlines, and it is precisely on top of a dome that the greatest confusion prevails. It is not clear whether variable compounding would help much in this case or not.

A condition in which the method might definitely decide an ambiguity is that of a hallow or a groove in an otherwise flat surface. The ordinary seismograph, showing the same dip on both sides and perhaps confusion in the middle, could not distinguish whether the confusion were due to a groove or a ridge. Variable compounding might decide. More especially, if the surface were wavy or corrugated, this method might pick out the grooves where the ordinary method would only show confusion all along the line.

The most startling claim made for the new method is that it can reveal the presence of a fault by detecting the wave scattered by its edge, and can distinguish this wave from a regular reflection coming from a lower bed and arriving at about the same time. Everyone will agree that such an ability would be a most valuable one, for it is characteristic of ordinary dip shooting that it can pass right over a fault without revealing it.

Fig. 5, at A, shows a condition for a vertical shot under which a scattered and a reflected wave will arrive at the geophone, G, at the same time.

Now first of all it may be said that it is doubtful whether this scattered wave can be detected at all on account of its feebleness. The theory is the same as that for the scattering produced by a small rigid sphere in the path of the waves. The fraction of the incident energy that is scattered by the sphere, varies as the fourth power of the radius, and inversely as the fourth power of the wave length. It therefore increases very rapidly as the size of the sphere is increased or the wave length is reduced. The formula can be found in Lamb's *Dynamical Theory of Sound* and elsewhere. This formula shows that a sphere will scatter practically all of the incident energy up to a wave length equal to three times its diameter. If it is assumed that 200 feet

is the maximum length of our useful waves, then a sphere of 35 foot radius would scatter practically all of the incident energy up to that wave length. It will also scatter a part of the energy of the longer waves.

If the sphere were reduced to a point, it would scatter no energy at all. Hence, if the edge of the fault were perfectly sharp, there would be no scattered wave. One must assume, therefore, that the fault has a rounded edge and that the radius of this quarter-round is at least 35 feet.

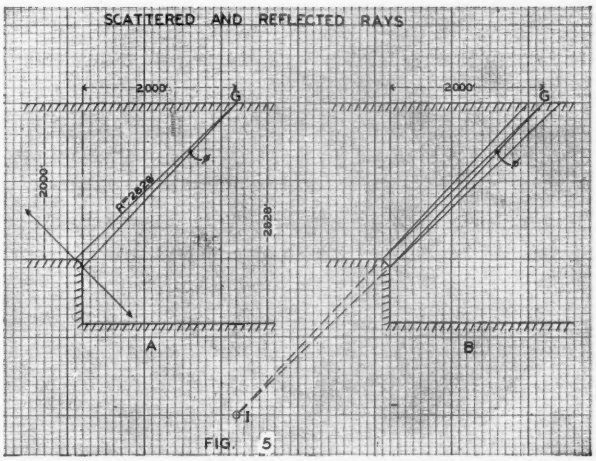


FIG. 5

Parenthetically it may be remarked that diffraction has nothing to do with this matter. Diffracted rays are those that are bent into the geometrical shadow in back of the sphere. Now, the size of the sphere has been so chosen, in this instance, that nearly all of the incident energy up to a wave length of 200 feet is scattered. Hence almost none of this energy is diffracted. Waves longer than 200 feet will be more and more diffracted as their length increases. If the sphere were smaller, less energy would be scattered, more diffracted. The diffracted rays pass on beyond the sphere. They are not thrown back. They do not reach the geophones except by subsequent reflection.

From the figure it is evident that all of the energy emanating from G that falls on the edge of the fault, is contained in the small angle ϕ. But this energy is scattered over an angle of 180 degrees.

Hence the proportion of it that is returned to an equal area at G is $\phi/180°$—in the case illustrated, 1/160.

Consider now what would happen if the rounded corner were replaced by a flat surface, as shown in Fig. 5, under B. This surface receives the same amount of energy as did the corner. But since the reflected wave now appears to come from an image point at twice the distance, the proportion of it that is returned to an equal area at G is $\frac{1}{4}$. The ratio of the energies returned by the scattered and the reflected waves is therefore 1/40. The same would be true if the reflecting surface were somewhere else at the same distance.

Matters are not quite so bad when one considers the amplitudes, for since the amplitude varies as the square root of the energy, the ratio of the amplitudes would be the square root of 1/40, or roughly $\frac{1}{6}$. But even at that, it is hard to believe that so small a wave, which is well below the general noise level, arriving simultaneously with one six times as big, could be detected by the compounding machine, or by any other method. And this applies to a fault whose edge is only 2000 feet down. At four times the depth, where faults become important, the amplitude ratio would be only 1/12.

But suppose for the sake of argument that the small wave could be detected. One does not yet know whether it is a scattered or a reflected wave. It is claimed that one can distinguish them by reason of the greater wave front curvature of the scattered wave.

Now, the curvature of the wave front shows up on the ordinary record as a curved step-out line. It cannot be detected on the analyzed traces. In fact, any distinct curvature would hinder the proper working of the machine by preventing the signal from ever coming into exact phase at all points of the straight slit. The ordinary records would therefore have to be used.

Now this curvature is in general very slight. It diminishes as the depth or the step-out increases. It is therefore a maximum for zero step-out, that is, when the image point lies directly under the middle geophone. If one measures the curvature by the bulge of the step-out line at its middle point, then, for an image point 2000 feet below the middle geophone, this bulge is one one-hundreth of a second—the space between two time lines. For all greater depths, and for all other step-outs, the amount is less, usually much less. In the majority of cases, therefore, the curvature cannot be detected at all, much less distinguished from some other curvature.

Of course the edge of a fault is never so beautifully rounded or so

straight as has here been assumed. Consequently the scattering will not be uniform, but will be greater in some directions than in others. It is just possible that one of these strong spots might accidentally be picked up, and the scattered wave actually detected. But the stronger the spots, the fewer they are, and the less likely it is that one will be accidentally picked up. Furthermore, a greater concentration of energy in one direction can only be due to a flatter curve at the spot where the reflection takes place. The flatter curve acts less like a scattering particle and more like a convex mirror. The flatter the curve, the further the image point lies behind it, and the more the supposed scattered wave looks like a plain reflection. Hence, the more detectable the scattered wave, the less distinguishable it is from a plain reflection. It does not appear, therefore, that a scattered wave, even if detected, could be distinguished by means of its greater curvature from a reflection. If a fault can be detected at all, it can only be so shallow a one as to be of no importance.

The final conclusion of this paper is—that the multiple hook-up system is better adapted to easy than to difficult regions, while conversely, the method of "controlled directional sensitivity" is superior only in certain special and difficult cases which, however, are not very many nor always very important.

DISCUSSION

FRANK RIEBER: Dr. Mott-Smith has devoted some time and thought to a consideration of the analyzer, and has stated as his conclusion that he feels that the method is applicable only to a certain narrow range of problems. He seems to have the impression that the earlier and more widely used methods are better adapted for the general run of work.

He has reached this decision by seeking for and pointing out what he believes to be the adverse effects of the analyzer principle. In his absorption with these apparent drawbacks, he has overlooked certain very apparent advantages inherent in the method and hence his conclusion that it is not well adapted for general use may be a little premature. No ledger is ever balanced exclusively in red ink.

The method used by Dr. Mott-Smith is one formerly very popular in the school of classical physics. He has chosen a simple wave form, which he believes to be comparable to a reflection. He has then laboriously reproduced, with pencil and paper, what he believes the analyzer would do to such a wave—and he then criticizes the results of his efforts as if he were criticizing an actual analyzed record.

To my certain knowledge he has never seen an analyzer in operation. He is under the further disadvantage of not having followed, over a considerable period of time, the development of the technique by which analyzed records are read and interpreted. Therefore, when he points out a strange wave form on the diagram which he has made, he may not realize that such a wave is thoroughly familiar to an expert reader of analyz-

er records and is automatically discarded or given its proper rating on account of its very distinctive appearance. His criticism, in other words, seems to be more a bewilderment at a strange form of record than a proof that such a record cannot be used effectively.

Specific evidence of his unfamiliarity with the analyzer appears in his paper, where he goes to some pains to point out the adverse effects of an analyzer with a single slit, set at various slants across the film. These criticisms are quite correct and well taken—but they do not apply to the analyzer as it is built. The actual instrument contains a series of slits, none of which slants across the record. In this and other similar cases, Dr. Mott-Smith has been greatly handicapped by his lack of specific knowledge of the machine on which he is offering specific criticisms.

The modern approach to physical investigations does not proceed entirely on theory. A limited theoretical discussion is sufficient to point out and define the field to be investigated. Practical results obtained by experiment are thereafter more valuable than further theorizing. Frequently they uncover unsuspected things, never dreamed of in the preceding theory, which must be explained later.

A simple case in point is the discussion which once raged over the curved flight of a baseball. Certain theorists contended hotly that it was an optical illusion, and proved to their own satisfaction at least that no physical principle was involved and that the ball only seemed to curve. A practical experimenter demolished this point of view very simply by placing three posts in a line, and allowing an expert pitcher to weave the ball back and forth through the posts. The ball actually curved. No one could doubt it, and proceeding from the safe advantage of an experimentally proved fact, the theorists were able to deduce the reasons for the curvature, and to locate flaws in their previous assumptions and reasoning.

Turning specifically to Dr. Mott-Smith's criticisms, I find that he does not believe that the direction of arrival of a wave can be ascertained with the analyzer to the same degree of accuracy possible with the earlier visual records.

Actual experience in the use of the analyzer indicates an accuracy at least comparable with other methods, and it is reasonable to suppose that this would be so, since both depend on the same fundamental recorded waves, but differ only in the method of ascertaining the direction in which they line up.

For those who are interested in detail, C. H. Johnson has prepared a brief discussion of the relative accuracy of the two methods, from which it will be seen that they are quite comparable. For such a proof it is not necessary to go beyond some very simple assumptions and a small amount of geometry.

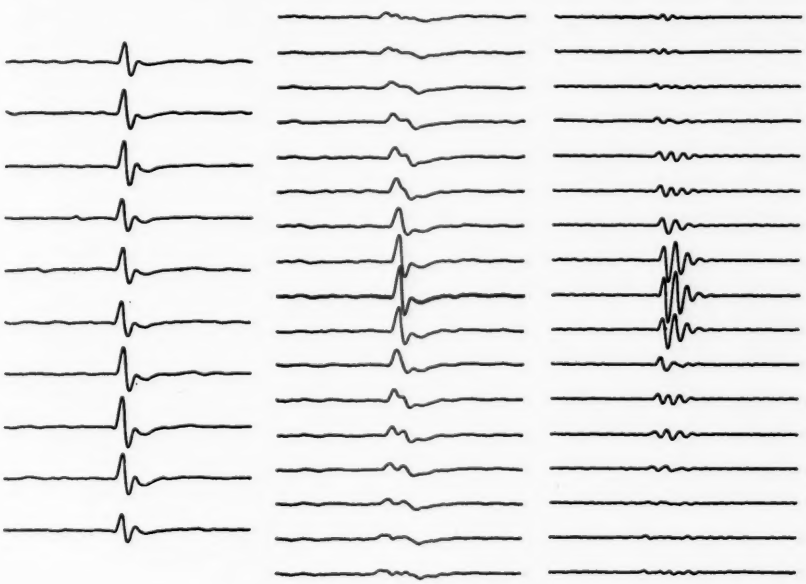
Next, I find that Dr. Mott-Smith fears that machine noise in the analyzer may be added to an already overburdened vibration record, thereby confusing the issue and rendering the identification of waves more difficult. Unfortunately, he has not clearly defined what he means by machine noise. Two possibilities seem open.

First, he might mean the type of noise referred to in the motion picture art as machine noise—that is, vibrations generated in the reproducing apparatus itself, and not related in any way to the original recorded waves. If this is what is disturbing him his fears are groundless. While the analyzer does introduce a small amount of noise while reproducing records, it is so small that it is not detectable by eye. Ample proof of this is given, for those readers interested in details, in the discussion by C. H. Johnson.

If, on the other hand, Dr. Mott-Smith means, by the term "machine noise," the disturbed wave forms producible by the analyzer and similar to the stretched out waves

he has obtained in his graphic analysis, this is another matter. The analyzer, in its simplest form, will admittedly produce such peculiar waves. The question to be raised is whether such waves interfere seriously with the operation of the method.

Our experience shows that they do not, and that a trained operator is readily able to distinguish an actual reflection, properly cumulated, and to discard these distorted waves on account of their form and position on the record. Hence, while they may



A B C

A. Record of the individual geophone sound tracks taken from a synthetically prepared film by the analyzer.

B. Cumulation by the analyzer of the sound tracks used to produce A.

C. Same as B, but with analyzer circuit modified.

obliterate a small part of the record and prevent the recognition of other waves at precisely that place, they are not, of themselves causes of erroneous conclusions.

Experience has further taught us something that is also well known to any practical geophysicist—namely, that certainty in the conclusions which can be drawn from geophysical evidence arises out of the cross combination of many individual observations. No single observation with any system should ever be relied on for accuracy within extremely narrow limits of error.

Furthermore, Dr. Mott-Smith assumes that the operation of the analyzer is limited to achieving the same results which he has achieved with a pencil. That this is definitely

not so will be seen from the two illustrations below. Both of these illustrations, by the way, were actually made with the analyzer itself, and not with a graphic construction. Furthermore, the wave form used was closely similar to the waves we are actually recording from the earth on our film. We have not adopted an arbitrary two-loop sine wave for this demonstration.

The first record is the way the single tracks were reproduced by the analyzer.

The second record was made with the analyzer performing merely as a device for adding the wave trains at various ΔT settings. The location of the maximum is readily seen, and also the disturbed wave forms on either side of this maximum are quite evident.

The third record was likewise made on the same analyzer and from the same film. The analyzer circuit was altered, however, to eliminate these distorted wave forms electrically. The increase in clarity is at once apparent. This is a possibility which Dr. Mott-Smith has evidently not considered at all in his reasoning, but it nevertheless constitutes a very definite advantage for the analyzer method.

I find, further, from this paper, that he has certain fears as to the reaction of the analyzer to small waves. On this point, however, he does not seem to have decided fully on which side of the fence he intends to stand. In one place in his paper I find that he believes that diffracted waves, if present in the earth, would have very small amplitude and hence could not be recognized on the analyzer. And in another part of his paper, I find the fear expressed that small accidental vibrations, having no geologic significance, will be, nevertheless, accentuated and brought out by the analyzer. I shall attempt to define my own stand on this point later by stating concisely what I believe, and what experience has shown to be true, regarding the magnitude of the detectable waves that can be safely accepted as having significance.

Before doing this, however, I wish to deal with the matter of diffracted waves, which also seems to cause Dr. Mott-Smith some concern. As I gather his point of view, he seems to feel that the term "diffraction" is a misnomer. This is purely a question of terminology. As long as we all call a wave train by the same name when we are talking about it, it makes little difference what that name may be. But I still rise to defend my choice of terminology, using the customary appeal to authority. The two illustrations shown below are taken from the work of Professor Vern O. Knudsen,[1] and show waves similar to those which I have termed diffractions. And the term used by Professor Knudsen to describe the waves is "diffracted waves."

Passing from these relatively minor points to the more serious claim, namely, that diffracted waves could not possibly contain sufficient energy to be recorded, this does not seem to be in accord with our experimental experience. We first became interested when we discovered in our actual field records waves which appeared to originate in faulted zones, which could not readily be explained as simple reflections.

The possibility of diffraction suggested itself as an explanation. Again we experimented, by developing the machine we have termed a Strobograph, with which we took the picture reproduced in Plate F, which was first shown in connection with an earlier paper.[2] In this illustration, there appears a wave which is certainly not a simple

[1] Vern O. Knudsen, "Architectural Acoustics," pp. 52 and 54, John Wiley & Sons, 1932.

[2] Frank Rieber, "Visual Presentation of Elastic Wave Patterns Under Various Structural Conditions," GEOPHYSICS, Vol. 1, No. 2, July, 1936.

reflection, and which contains ample energy to make it clearly visible. Plate G indicates how some of these waves originate at the corners of the intervening object.

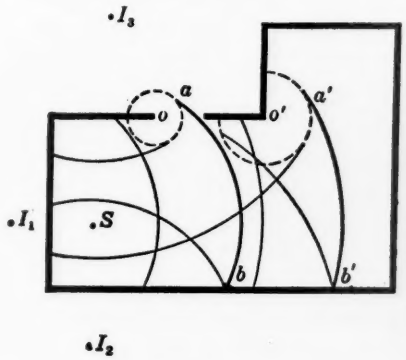


PLATE D

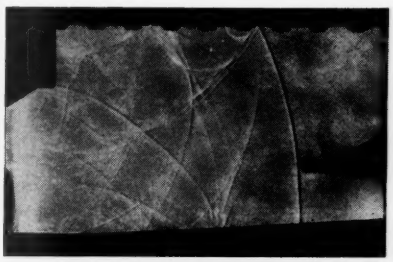


PLATE E

In Plates D and E, are shown almost identical waves, to which Professor Knudsen refers as diffracted waves. Since the term diffraction is used to describe a sound wave originating at the edges of a geometrical body, then the term may also be applied to describe a wave whose source is in a fault face.

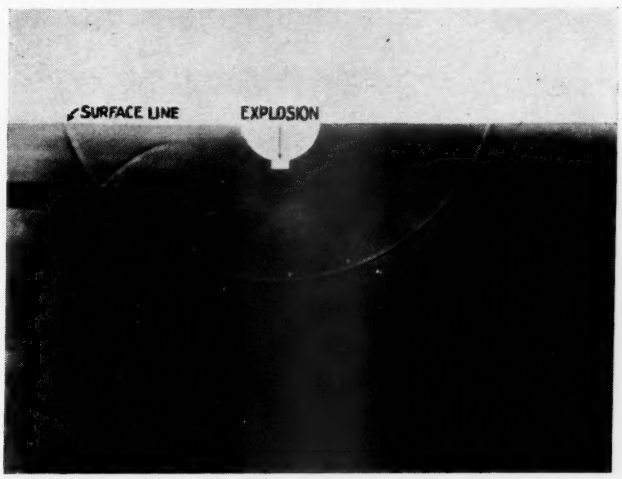


PLATE F

Dr. Mott-Smith has gone into an elaborate mathematical proof to demonstrate that the sharp edge of a faulted stratum could not send back to the surface of the earth enough energy to be registered by instruments. As opposed to his conclusion, we have the experimental evidence just adduced that waves *do* come back from sharp edges, and that they *do* contain a considerable amount of energy. It seems, therefore, that it

might be proper to investigate Dr. Mott-Smith's mathematical contentions to see whether possibly he has proceeded from correct assumptions which are applicable to the case at hand.

Coming now, in conclusion, to the matter of how small a wave may be before we distrust the geological interpretation given it, I feel that we should first remember that there is no absolute scale of magnitude which can be usefully applied in sound measurements. In other words, we are not concerned with how many ergs of energy may be contained in a vibration—but we *are vitally concerned* with whether it is bigger than or smaller than other vibrations which are recorded on the same record.

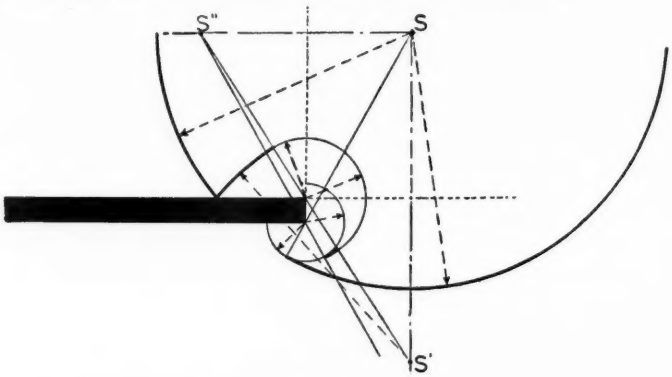


PLATE G

Anyone with the slightest experience in reflection shooting realizes that the earth is continually in vibration from wind and other causes, and that these vibrations will be picked up and recorded by the apparatus. We can, if we wish, term the average amplitude of such vibrations zero level, and refer all other vibrations on the record to this level for comparison, using any scale we wish, preferably the conventional unit adopted in sound measurements, the decibel.

These background vibrations are different in magnitude every day and every hour. To discern safely a reflected wave, superposed on this background, the magnitude of this wave must stand out above the background noise by a satisfactory amount for safe picking. Provisionally, let us say that this might be, as a minimum, double the amplitude of the background noise, or 6 decibels up on the scale of amplitudes.

A "small wave" would, therefore, be one double the amplitude of the background noise, and this would certainly be the least amplitude with which the apparatus is called on to deal. If it can safely and accurately identify such waves, it is a satisfactory equipment. Measured by these standards, the sound records and analyzer reproduction used in the Sonograph system are certainly quite satisfactory, in that it is readily possible to identify and use small waves only 6 decibels above the normal noise level. As pointed out by Dr. Mott-Smith, variable compounding emphasizes the difference between the reflected wave and the background noise, thereby making it easier to pick small reflections from the analysed record than from a corresponding single-trace record.

Due to the principle of the analyzer, which takes into account not only the magnitude of the individual recorded waves, but also the perfection with which they line up

with each other, it has been repeatedly found possible to identify on an analyzed record waves not remotely visible on the visual form because of their small magnitude.

Therefore, if diffracted waves do return from the earth with a magnitude of 6 decibels greater than background earth noise, we can and do detect and use them in our interpretation.

One more very minor point occurs to me in connection with diffraction. In the discussion of my paper[3] at the last Houston meeting, I was asked whether a diffracted wave differed from a reflected wave, and, if so, in what respect? My reply was that a diffracted wave had greater curvature than a reflected wave arriving at the same time.

For some strange reason, Dr. Mott-Smith gained the impression that we expected to detect this curvature from a single sound film taken with a short receptor spread. He has shown, very properly, that this could not be done within the limits of error of the operation of the machine. In our work, however, we have never attempted to detect such curvature from a single setup—and never claimed that it could be done. Two adjacent instrument setups are required, since the curvature of a sphere of such radius requires observation of a considerable portion of the wave front.

I now wish to consider the general conclusion which Dr. Mott-Smith has drawn —namely, that the sound film and analyzer technique has only a narrow range of practical application. In replying to his paper I would gladly have dealt with this conclusion alone, and omitted all detailed references to specific criticisms of the method, except for the fact that reticence on my part might imply that I was in agreement with him on the points he has brought up.

To decide the larger matter of the general applicability of the method, I feel that we should consider its fundamental advantages as well as its alleged disadvantages. I can sum up these advantages in the following statement. By recording events in the field in their entirety, and as a sound film, we can collect all the physical evidence that the returning waves can bring us and retain it in a permanent form. We may thereafter take such a film to the laboratory and repeat the shot, in slow motion, for careful study.

In this repeating process, we may make all of the adjustments which, with other systems, must be carried out in the field. We can, for example, use different filter settings, or we can reproduce the record with any desired amplitude.

With earlier forms of apparatus, the operator shot time after time in the same hole, experimentally setting his filters and his amplitude in an attempt to bring out the clearest type of record. If he was fortunate, the hole did not cave in until he was through experimenting. Otherwise, a new hole was drilled at additional expense, and the conditions of the experiment were thereby repeatedly altered until good records were obtained.

And regarding amplitude—the visual record, to be satisfactory, must not have waves so large that they tangle with their neighbors, or so small that they are not clearly evident to the eye. These two limits constitute a relatively narrow range which must be achieved once and for all by adjusting the size of the shot and the setting of the amplifiers. While automatic volume controls and other devices have minimized this difficulty for present day field work, they have not eliminated it.

A record on film, however, has a much higher permissible latitude, in that we are limited only by the range of blackness and lightness producible on the emulsion, rather than by the ability of the eye conveniently to identify waves. It is seldom necessary to

[3] Frank Rieber, loc. cit.

repeat shots and make adjustments of amplitude when recording on sound film. This, again, reduces the hazard of caving in of the hole before the shots of any one setup are finished.

Summing up, although the process of recording on film and reproducing in the laboratory is slightly longer and possibly a little more expensive than the earlier methods, it is amply justified by the ability to repeat the shot in the laboratory as often as one wishes—and as long after the field work as one wishes—and under any chosen conditions of filtration and amplitude. This advantage alone would seem to warrant the claim that the Sonograph is not limited to special cases, but has definite advantages for general work.

When we add to the above advantages that of directional selectivity, the new method seems well worthy of a place in the general armament of geophysical equipment, particularly as attention is being devoted more and more to the details of structure, rather than to their general position and dimensions.

Dr. Mott-Smith seems still to doubt the fact that the analyzer can unscramble confused waves and identify them separately, where the eye would be unable to do the same thing on a visual record.

Our practical experience in actual geophysical exploration, however, has shown repeatedly that the apparatus does possess this advantage to a very high degree. However, a very extensive experimental proof of this fact was given by me in a previous paper[4] and it does not seem worth while to pursue the subject further here.

After all, the field performance of a method is the only final proof of its usefulness. Such proof is rapidly accumulating, and most of it is of such a nature as to dispel any doubts that Dr. Mott-Smith may have regarding the advantages of the Sonograph.

C. H. JOHNSON: Though Dr. Mott-Smith mentions the fact that the directional sensitivity depends as much upon the geophone spread as upon the measurement of the differential time of arrival, he ignores this fact in his comparison between the requirements of "multiple hook-up and variable compounding." He likewise ignores this fact in the conclusions he draws from various published descriptions of Analyzer performance. Thus, we may shorten this discussion by overlooking his treatment of directional sensitivity and considering the problem anew from its fundamentals.

The effective direction of arrival may be determined with sufficient accuracy from the relation

$$\sin \psi = \frac{V\Delta T}{B}.$$

where ψ = the angle of arrival,
V = the mean velocity over the path,
ΔT = the difference in time of wave arrival at the two ends of the geophone spread,
B = the geophone spread.

The choice of geophone spread is a problem common to all present forms of dip shooting. The Sonograph and the usual type of seismograph are equally flexible in this regard. Choice of velocity function in likewise a problem handled in the same manner by both systems. A comparison of "directional sensitivity" thus resolves itself into a comparison of the accuracy of determining ΔT.

[4] Frank Rieber, "Complex Reflection Patterns and Their Geologic Sources," GEOPHYSICS, Vol. II, No. 2, March, 1937.

As a basis for the following treatment of ΔT accuracy, it is necessary to correct a possible misinterpretation of one of Dr. Mott-Smith's remarks. Where he compares the picking of a maximum unfavorably with the picking of a null, one might infer that ΔT is determined on the usual seismogram by locating nulls. Actually, this is not the case, the arrival of a wave on a visual seismogram being denoted by a peak or a trough near the beginning of the wave. Thus, the visual seismogram suffers equally with the analyzed sonogram as regards the necessity of picking a "maximum."

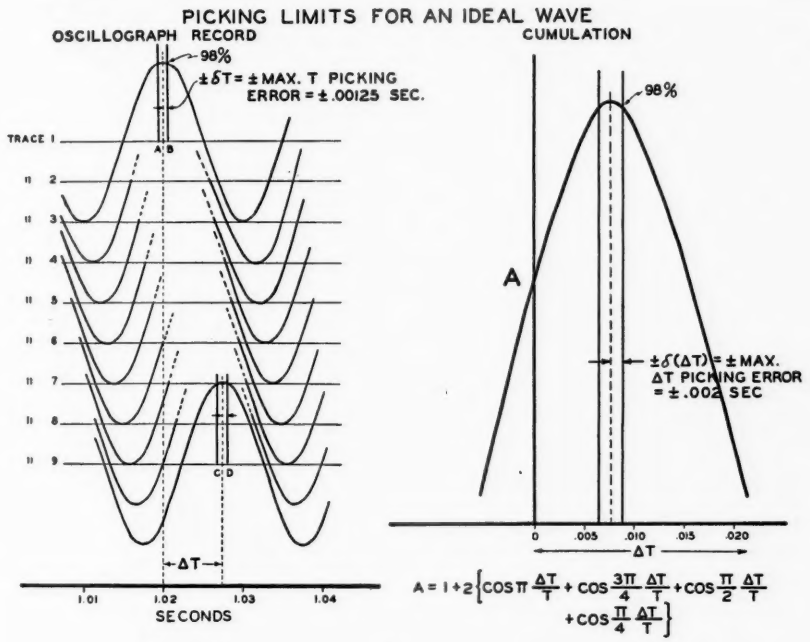


FIG. 1

Fig. 1 shows an idealized wave on a visual seismogram on the left and a portion of the "Amplitude vs. ΔT function" from an analyzed Sonograph record of the same idealized wave. In the equation represented by the curve on the right, derived for steady state,

A = Amplitude after analysis,
ΔT = Differential time of wave arrival at the two ends of the geophone spread,
T = period of the wave.

On the right of Fig. 1, it should be observed that ΔT is the difference between the positions of the two maxima. On the left of Fig. 1, ΔT is the actual position of the maximum.

It seems a fair basis for comparing the accuracy of location of two such similarly shaped types of maxima to assume that either may be picked in error by a slight

amount deviating from the true maximum by the same per cent of the maximum amplitude. In the figure this has been shown as from a point 98% of the maximum on one side to a point 98% of the maximum on the other side. Actually, either maximum could be picked with a half or a third this error, 98% being chosen for clear illustration.

Assuming the 98% figure for the sake of argument, let us determine the magnitude of the maximum ΔT error in each case, remembering that the ΔT errors may fluctuate from zero plus and minus to these extremes.

The drawings have been made for a 50-cycle wave. Thus the error in locating the maximum on trace 1 at the right side of Fig. 1 is found to be (for the assumed 98% limits, of course) $\pm .00125$ sec. or from A to B. On trace 9 the error is likewise $\pm .00125$ sec. or from C to D.

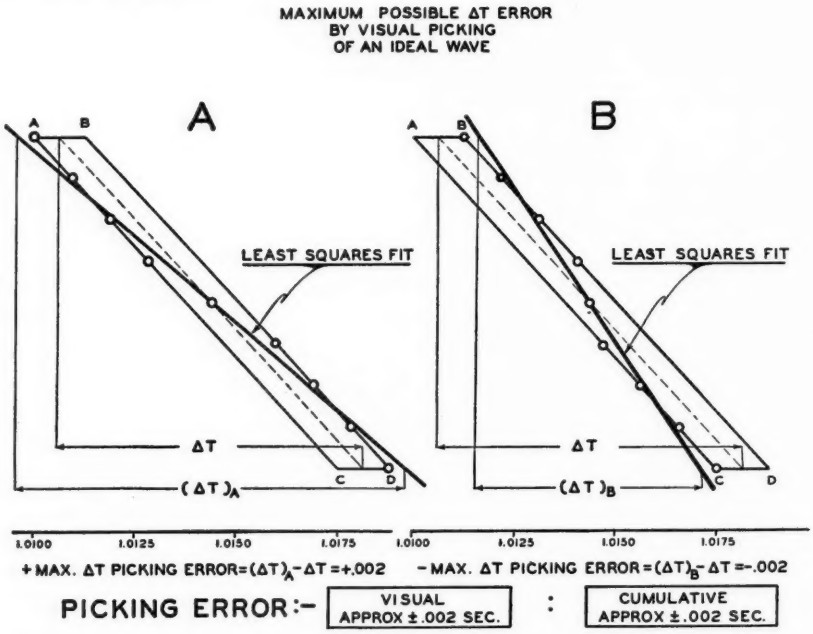


FIG 2

Referring now to Fig. 2: The region A–B C–D has been enlarged along the time axis in order to make it clearly visible.

Possibly at this point it should be remarked that in determining ΔT visually, it is not safe to put the entire responsibility on one trace, for unlike the idealized Fig. 1 most waves are not entirely uncomplicated by extraneous vibration which may alter the apparent phase on some of the traces. Thus the concensus of opinion of all the traces must be used. Remember that the error region has been greatly enlarged in Fig. 2. The visual picker is probably seldom aware of the exact grouping of errors of the various traces. However, they do exist and the final ΔT line accepted for the wave is his best visual adjustment of the errors. Since visual adjustment of errors will

vary from individual to individual, let us assume that a least squares fit represents the ideal visual adjustment of errors.

Since we are attempting to determine the maximum ΔT error by the visual and the analyzer methods, the worst arrangement of ΔT errors on the individual traces is shown in Fig. 2. At the left, traces 1 to 4 have the maximum minus ΔT error, traces 6 to 9 have the maximum plus ΔT error and trace 5 has no ΔT error. These circumstances give the maximum plus ΔT error between the best fit line and the true line. Its magnitude is +.002 sec. In a similar fashion at the right of Fig. 2 the maximum minus ΔT error is found to be −.002 sec.

Returning to the right hand side of Fig. 1, the ΔT error is seen directly to be ±.002 sec.

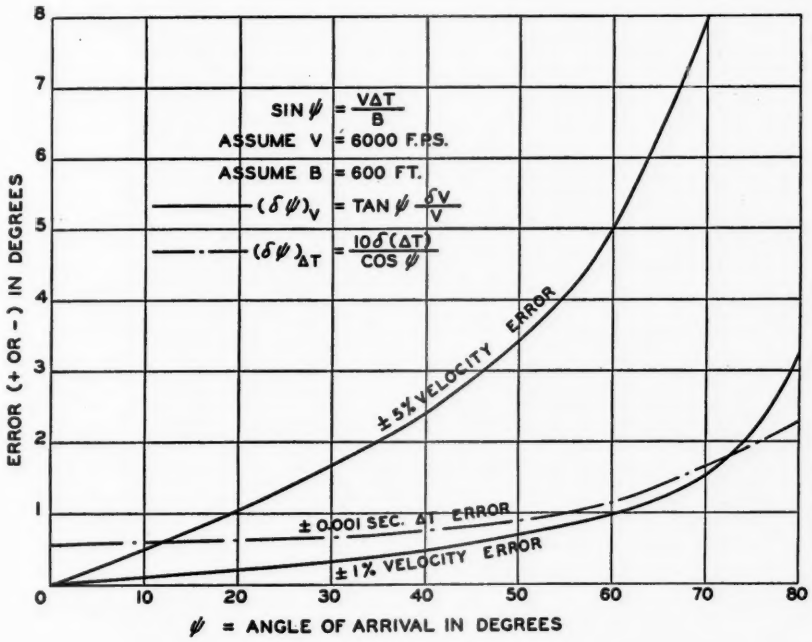


FIG. 3

Thus for the ideal case, the accuracy of ΔT *determination is the same for the visual and the Sonograph Methods.*

It is of interest to investigate Dr. Mott-Smith's statement that all other factors in dip shooting may be measured more accurately (in terms of the effect upon angle of arrival) than ΔT.

Fig. 3 shows the error in angle of arrival as a function of angle of arrival for an assumed velocity of 6000 feet per sec. and an assumed geophone spread of 600 feet.

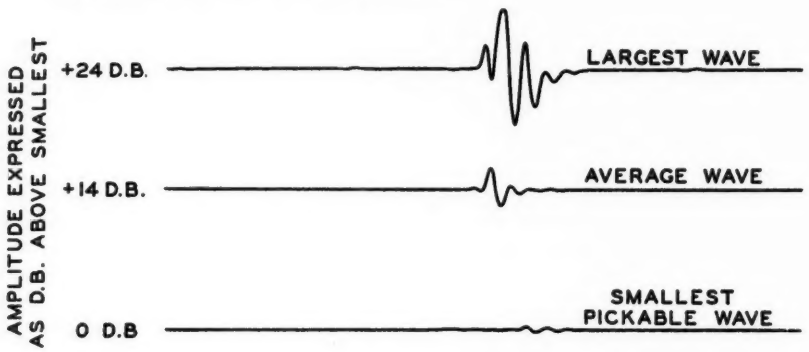
Where velocities may be determined by shooting in wells, the accuracy in velocity determination may approach or even better 1%. Where such shooting cannot be done,

the error may easily approach 5%. Accordingly, curves have been drawn from both 1% and 5% velocity errors.

The error introduced by an error of .001 sec. in ΔT is also plotted, this figure being a probable maximum error for both the visual and the Sonograph methods.

Velocity errors are seen to be more important at large angles and ΔT errors more important at low angles. The error due to ΔT errors is seen from the equation to be inversely proportional to B, hence, if the geophone spread were increased to 1200 feet, the error in angle of arrival due to .001 sec. ΔT error would be half that shown on the chart.

Incidentally, this chart shows the effect of errors in measuring the geophone spread. Within narrow limits, a 1% and a 5% error in the measurement of the geophone spread will result in the error curves shown for a 1% and a 5% velocity error. Normally, errors in geophone spread are, of course, negligible.



CONSTANT AMPLIFICATION
20 D.B. BELOW MAX.

FIG. 4

In his abstract and again in his paper, Dr. Mott-Smith speaks of "artificial noise, which is produced by the machine itself." A careful reading of his paper makes it clear that he means by this the "stretched wave" remnants or "rumble." In many places his abstract will appear without his paper and hence a mistaken impression may be created that the analyzer mechanically or electrically introduces vibrant energy not present in the original record. To rectify any such mistaken impression, the following figures are presented to show the noiseless operation of the analyzer.

Fig. 4 represents the reproduction of an arbitrary wave at a normal amplification setting on the analyzer. For the purposes of illustration, something slightly less than the largest wave frequently utilized was shown as the largest wave. This discussion is principally interested in smaller and smaller waves. So this approximation is of no consequence. It is seen that the "smallest pickable wave" is not usable at this amplification, and on the ordinary visual record would be discarded or a second record shot at higher amplification or with a larger charge of dynamite.

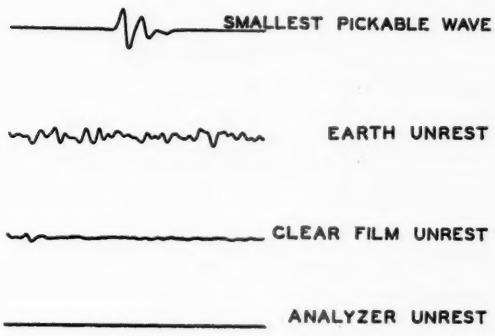

SMALLEST PICKABLE WAVE
EARTH UNREST
CLEAR FILM UNREST
ANALYZER UNREST
MAXIMUM AMPLIFICATION

FIG. 5

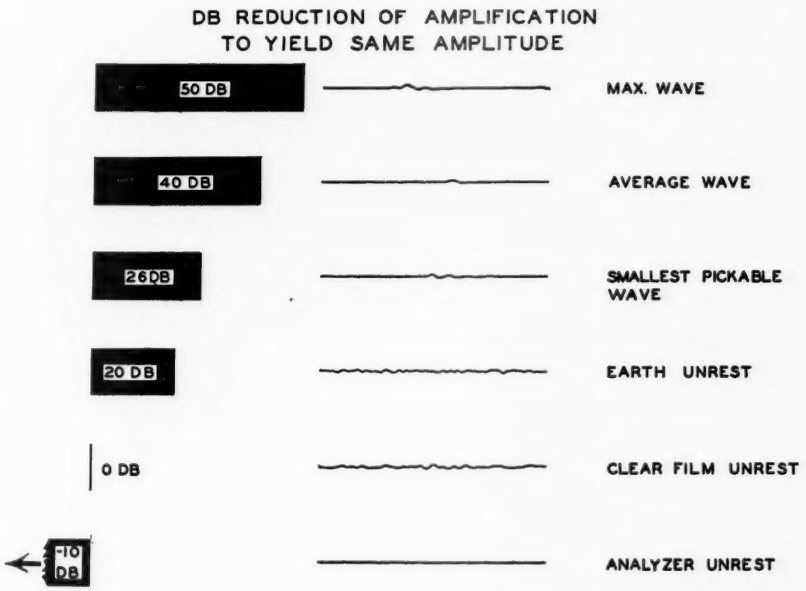

DB REDUCTION OF AMPLIFICATION
TO YIELD SAME AMPLITUDE
50 DB
MAX. WAVE
40 DB
AVERAGE WAVE
26DB
SMALLEST PICKABLE WAVE
20 DB
EARTH UNREST
0 DB
CLEAR FILM UNREST
ANALYZER UNREST

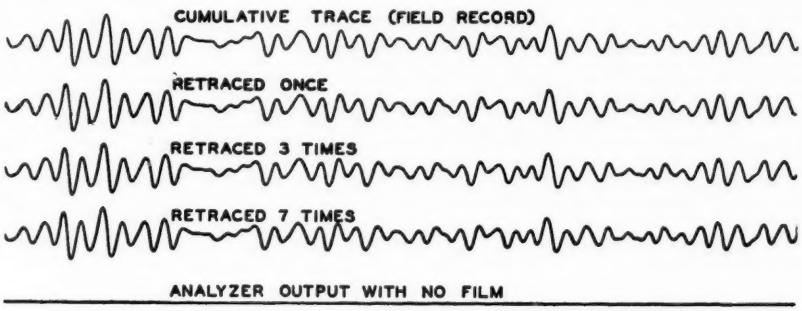
FIG. 6

On the analyzer, however, the amplification is merely increased 20 db with a turn of a knob, and the "smallest pickable wave" becomes of the useful size shown in Fig. 5. At this maximum amplification other small waves appear. Normal earth unrest is about half the size of the "smallest pickable wave"—in fact, that is what determines the size of the "smallest pickable wave," which may actually be of the same magnitude as the earth unrest and still be salvaged by analysis which will reveal it above the earth unrest by its rise and fall in size as ΔT is varied through and beyond the time at which the small wave appears. Fig. 5, however, is particularly conservative.

Below earth unrest (common to Seismograph and Sonograph alike) appears a trace of unrest attributable to small scratches and unobserved dirt on the Sonograph film. This is seen to be negligible compared to normal earth unrest.

Below clear film unrest is what may be termed mechanical or electrical analyzer unrest. Even at the maximum amplification it is absolutely zero.

The above two figures have been summarized in Fig. 6 in a slightly different form. Here the amplification has been reduced for each type of wave, until it has reached the amplitude of clear film unrest as disclosed at maximum amplification. The true "analyzer unrest" would require an increase rather than a reduction of amplification to bring it to this level; so, its "reduction of amplification . . . " has been estimated at minus 10 db or more.



REPRODUCIBILITY OF ANALYZER OUTPUT

FIG. 7

Fig. 7 is perhaps a more striking demonstration of the noiseless and reproducible quality of analyzer traces. The retraces represent a complete reanalysis at the same ΔT setting and show only the normal widening of the line to be expected from the repeated application of ink.

Clearly, "artificial noise, which is produced by the machine itself" is shown to be absent or negligible.

REVIEW

Geophysical Investigations in the Emreged and Submerged Atlantic Coastal Plain. Part I. Methods and Results. By Maurice Ewing, A. P. Crary, H. M. Rutherford and B. L. Miller. Bulletin of the Geological Society of America, vol. 48 (1937), pp. 753–802.

"This report describes the development and application of a seismic method of procedure for making studies of submarine geology. Any new tool for gaining information about the submerged portion of the earth's crust would be noteworthy because few such tools are available. The present method merits additional consideration because its findings are directly translatable into such concrete terms as to depth to certain horizons in the rocks and the elastic constants of the rocks." This paper describes the methods employed and results obtained in making a series of refraction measurements at four stations at sea south of Woods Hole, Massachusetts and a fairly complete section across the emerged and submerged portions of the Atlantic Coastal Plain at Cape Henry, Virginia. Employing conventional methods and apparatus the authors shot refraction profiles along a line extending east from Petersburg, Virginia, 150 miles to the edge of the continental shelf, during the period from May to October, 1935. In addition to the refraction profiles, several reflection stations were shot. In some cases the shots and detectors were placed as deep as 280 feet below the surface of the water.

The water work differs essentially from seismic prospecting work in shallow lakes where the detectors have been planted securely in the mud by the use of long poles and may be considered the first submarine seismic work. The cases of the detectors were so designed that they could stand upright on the bottom after being lowered by means of cables. In all of the submarine refraction work the authors employed duplicate detectors lowered on opposite sides of the boat, the purpose being to discriminate between true ground motion and wave disturbances transmitted down the cables. Additional traces were recorded with lower amplification, using the same detectors. This was to assist in picking the arrival of the wave which had traveled directly through the water. In addition a detector was suspended a few feet below the surface of the water to record the water wave.

The most striking feature of the refraction work was the good use made of phases other than first kicks. " 'Later Phases' are impulses that arrive at the recording point subsequent to the arrival of one, or more waves. A later phase is recognized on a seismogram by a change in amplitude, frequency, or phase of the recorded motion. Frequently, later phases show only vaguely on seismograms, so that they cannot be identified with the desired degree of certainty. In the present work, certain later phases were so phenomenally distinct that they could be identified positively and timed precisely. Most of the travel-time curves bear many points, which are derived from later phases, especially on the U branches. This favorable circumstance permitted the necessary data to be obtained from a much smaller number of shots than would have been otherwise needed." This added information increased the accuracy of all determinations and on a number of profiles permitted the determination of a layer never represented by a first kick, so-called masked phases. "The depth (at Fort Monroe, Virginia) to the crystalline rocks, calculated from the seismic measurement, is 2260 feet. The depth to crystalline rocks found in the deep well drilled inside Fort Monroe in 1902 is 2243 feet. The agreement is perhaps partly fortuitous, and the error less than the probable error of the seismic determination, in view of the poor quality of the data available. There

would be a large discrepancy between seismic result and well log, if the *S* phase, which is masked here, had been overlooked."

Also of great interest is the definite identification of a wave reflected from the surface. This phase corresponds to a wave refracted in the crystalline rocks, reflected from the surface and again refracted in the crystalline rocks. The velocity and time check very closely except in one instance.

A comparison of land and water work is interesting since the disturbing "ground roll" which was prominent on the land records is entirely lacking on the water records. The average frequency at sea stations is said to be about twice that at land stations. This may account for the fact that the reflection records were disappointing on land but excellent at sea. "A striking feature of the reflection results is the great number of reflecting surfaces revealed." This contrasts with the three refracting layers observed. "Some of the identification of phases on the seismograms as reflections may be open to question, but many of them are uncommonly good. It is concluded that there are numerous horizons in the semi-consolidated zone at which slight variations in composition or lithification of the sediments are sufficiently abrupt to produce measurable reflections of the elastic waves." However, although many reflections were obtained within the sedimentary layer, no reflections seem to correspond to the interface between the sediments and crystalline stratum, where the velocity more than doubles.

The authors believed that the reflections obtained were good enough to map structure but in their work the stations were too far apart, and the reflections too numerous, to permit correlation.

This very interesting work was limited by the depth at which the equipment was capable of working. Some of the lines were shot in less detail than would have been desirable because the supply of dynamite was rather low. It is to be hoped that further funds will be made available for continuation of this work which is at present of scientific interest and which, may lead to developments that will in the near future, be commercially valuable in prospecting in coastal waters.

W. M. RUST, JR.

PUBLICATIONS OF THE SOCIETY OF EXPLORATION GEOPHYSICISTS

The following is a list of publications of the Society of Exploration Geophysicists, together with dates of publication and nature of contents. * Out of Print.

Year *Publication*

*1931 *Paper No. 1, "A Universal Slope Chart for Seismic Method of Geophysical Prospecting (Refractions)."* By O. C. Lester, Jr., and S. W. Wilcox was published in mimeograph form in February, 1931 as part of a group of papers presented before the Society in 1930.

*1931 *Paper No. 2, "A Suggested Method of Approach for Determination of Salt Dome Overhang."* By O. C. Lester, Jr., was published in mimeograph form in 1931 as part of a group of papers presented before the Society in 1930.

*1931 *Vol. I, Transactions of the Society of Petroleum Geophysicists,* was published in December, 1931 by the American Association of Petroleum Geologists and contained papers presented before the Society of Petroleum Geophysicists at the Annual Convention of the American Association of Petroleum Geologists at San Antonio, Texas, March 21, 1931; reprinted from the Bulletin of the American Association of Petroleum Geologists, Vol. 15, Numbers 11–12, November and December, 1931.

*1932 *Vol. II, Transactions of the Society of Petroleum Geophysicists* was published in March, 1932 by the American Physical Society and contained papers presented before the Society of Petroleum Geophysicists at the Joint Session with the American Physical Society at New Orleans, December 29 and 30, 1931. It is reprinted from Physics, March, 1932.

*1932 *"Semiportable Alternating-Current Susceptibility Meter"* by William M. Barret was published in Physics, Vol. 3, pp. 149–154, September, 1932 and was reprinted.

*1932 *Vol. III, Transactions of the Society of Petroleum Geophysicists* was published by The American Association of Petroleum Geologists in December, 1932. It contained papers presented before the Society of Petroleum Geophysicists at the Annual Convention of the American Association of Petroleum Geologists at Oklahoma City, March 25, 1932; reprinted from the Bulletin of the American Association of Petroleum Geologists, Vol. 16, No. 12.

*1934 *Vol. IV, Transactions of the Society of Petroleum Geophysicists,* was published by the American Association of Petroleum Geologists in January 1934. It contained papers presented before the Geophysics Division of the American Association of Petroleum Geologists in Annual Convention at Houston, Texas, March 24,1933; reprinted from the Bulletin of the American Association of Petroleum Geologists, Vol. 17, No. 1, January, 1934.

*1935 *Vol. V, Transactions of the Society of Petroleum Geophysicists* was published by The American Association of Petroleum Geologists at Tulsa, Oklahoma in March, 1935. It contained reprints from the Bulletin of the American Association of Petroleum Geologists; from the Transactions of the American Geophysical Union; from the American Journal of Science; from the Proceedings of the World Petroleum Congress; from Physics, of the American Physical Society; and from the Proceedings of the Physical Society of London.

1935 *Vol. VI, No. 1, Journal of the Society of Petroleum Geophysicists* was published by the Division of Geophysics, American Association of Petroleum Geologists in July, 1935. It contained many of the papers presented at the convention at Wichita, Kansas, in March, 1935. A very few copies left are reserved for members only at a price of $3.00. ($3.20 to foreign addresses).

1936 *Vol. I, No. 1, Geophysics*, A Journal of General and Applied Geophysics, was published by the Society of Petroleum Geophysicists in January, 1936. It contained many of the papers presented at the Mid-Year Meeting of the Society of Petroleum Geophysicists held at Dallas, Texas, in November, 1935. Vol. I, No. 2, June, 1936, and Vol. I, No. 3, October, 1936, continued the publication of papers presented at the Spring meeting of the Society, at Tulsa, and special technical papers submitted. Copies of all three numbers are available at $3.00 per number ($3.20 each to foreign addresses).

1937 Vol. II, No. 1, dated January, 1937, and Vol. II, No. 2, dated March, 1937, are now available; July and October issues will follow. All four issues may be had on subscription basis during the current year at $6.00 ($6.50 to foreign addresses) or the separate numbers at the regular price of $3.00 per number ($3.20 to foreign addresses). The separate number price will apply in all cases after Dec. 31, 1937.

The subscription price to GEOPHYSICS is $6.00 per volume to non-members, $4.00 to members of A.A.P.G., plus an extra mailing charge of 50 cents in either case to foreign addresses. After expiration of the year in which it is issued, each volume will be available only at the price of $3.00 per number ($3.20 to foreign addresses).

The Society is interested in obtaining copies of some of the papers and volumes mentioned above which are now out of print and will offer a *premium* for almost any of the numbers issued prior to July, 1935. Any person having such numbers and wishing to dispose of them will please communicate with the Secretary-Treasurer.

Send orders for back numbers or subscriptions to the Secretary-Treasurer, with check enclosed and made payable to the Society of Exploration Geophysicists.

M. E. Stiles, Secretary-Treasurer
2011 Esperson Building
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The Society of Exploration Geophysicists announces that, in the future, applicants for membership in the Society will be asked to submit the new sponsor forms properly filled out in addition to the membership application form. These new sponsor forms may be obtained from any officer of the Society.